Math 475 Homework 8

DUE APRIL 15TH, 2011

1. Find a quadratic function that models this data:

IN	OUT
0	-6
1	-1
2	-6
3	-21
4	-46
5	-81

2. Prove that the first difference of an Nth degree polynomial has at most degree $N-1$.

3. Prove an Nth degree polynomial has constant Nth differences. (Hint: use the last answer.)

4. We used the result in class that the Ath entry in the Bth row of Pascal's Triangle is the choose function $C(B,A) = \dfrac{B!}{A!(B-A)!}$. Let's prove it now. It suffices to show that $C(B,A)+C(B,A+1) = C(B+1,A+1)$, since this is the defining equation for Pascal's Triangle.

   (a) Check explicitly that this defining equation indeed holds for $A = 5$ and $B = 2$.

   (b) Prove the defining equation using the factorial definition of the choose function C.

5. Consider the function f such that $f(n) = 2^n$, for all $n \in \mathbb{N}$.

   (a) Compute $\Delta^i f(0)$, for all $i \in \mathbb{N}$.

   (b) Use the *Newton forward difference equation* to prove the sum of every nth row of Pascal's Triangle is 2^n.

6. Give a general non-recursive formula for every function $f : \mathbb{Z} \to \mathbb{R}$ that is equal to its own first difference function and prove there are no other such functions.

7. 5/10/11 and 5/12/11 will be days when you will present something cool that is math. You will have about 15 minutes. It will be graded on how cool it is and how much we learn from your presentation. Tell me:

   (a) With whom you plan to partner (size of teams should be between 1 and 3 people)

   (b) a specific idea (or two) that you have. Keep it short, but give enough detail so I can tell if it's a good or bad idea.

8. (*extra credit*) Define an infinite decimal expansion $0.d_1d_2d_3\ldots$ as the limit of a sequence of finite ("cut-off") decimal representations $0.d_1, 0.d_1d_2, 0.d_1d_2d_3, \ldots$.

   (a) Prove that the sequence in (8) is Cauchy. (Recall from Real Analysis: a sequence a_n is *Cauchy* if for any $\epsilon > 0$, there is a natural number N such that if $m, n > N$, then $|a_m - a_n| < \epsilon$.)

   (b) We define the real numbers, $\mathbb{R}$, as the *completion* of the rational numbers. Write down the definition of the *completion* of a set.

   (c) Write $0.999\ldots$ as a limit of a sequence of terminating decimals. Prove that the limit of this sequence is 1. Yes, you'll have to use an $\epsilon - N$ argument.