Math 475 Homework 4

Due February 18, 2010, 11:59pm

For the first three problems use only axioms, definitions and facts proven on worksheets. Please clearly cite anything you use. For the PEMDAS activity, you can use all arithmetic facts.

1. Prove natural number multiplication is associative.
2. Prove natural number multiplication is commutative.
3. Prove $(ab)^n = a^n b^n$ for all $a, b, n \in \mathbb{N}$

Order of Operations. What is "the order of operations"? It is a convention to interpret ambiguous arithmetic statements by assigning operations an order of evaluation.

In fact, one can always use matching parentheses to group arithmetic operations into unambiguous pairs. However, in practice this quickly gets annoying and it is hard to match large numbers of parentheses.

The standard convention for order of operations is to group expressions in the following order:

- **Parentheses**. Group every expression between a left and right parenthesis which contains no parentheses.
- **Exponents**. Group every pair of numbers directly connected by exponentiation. In strings where there are more than two such numbers in a row, evaluate the exponentiation from **right to left**. E.g. $2^{3^{5^7}} = 2^{(3^{(5^7)})}$.
- **Negative Sign**. Group a negative sign with the adjacent expression. E.g. $-x + 1 = (^-x) + 1$. Notably, $^-2^2 = {^-}(2^2) = {^-}4$, since the Exponents law came first. More importantly, $-x^2 = -(x^2)$.
- **Multiplication and Division**. Group any pair of numbers directly connected by $\times$ or $\div$. For shorthand, we will call this "evaluating the $\times$ and $\div$" and we will use similar terminology in the future. If more than two numbers in a row are connected by $\times$ or $\div$, evaluate them from left to right. E.g. $2 \div 3 \times 4 = (2 \div 3) \times 4$.
- **Addition and Subtraction**. Evaluate all the $+$ and $-$ from left to right, so $10 - 3 + 7 = 14$.

4. Using the *PEMDAS* convention above, re-write these expressions unambiguously with parentheses and evaluate them numerically.
   (a) $1 - 2 + 3 \div 4 \times 5 + 6 - 7$
   (b) $1 + 2 - 3 \times 10 \div 2 \times 9 \div 3$

5. There is no *mathematical* reason for the *PEMDAS* convention. (There are reasons of culture and tidiness of polynomials.) For instance, we could have declared the rule to be to evaluate all operations from left to right, or we could have given a different order of operations. In the problems below, I have given you some clues to figure out my secret new rules for evaluating ambiguous expressions. In each case, rewrite the expression with parentheses to get the target number and state clearly what the rules are. Make sure your rules can unambiguously evaluate any expression with numbers and $+, -, \times, \div$.
   (a) $1 - 2 + 3 \div 4 \times 5 + 6 - 7$ is $1\frac{1}{2}$ and $2 + 2 - 3 \times 10 \div 2 \times 9 \div 3$ is 15.
   (b) $1 - 2 + 3 \div 4 \times 5 + 6 - 7$ is $-1\frac{3}{16}$ and $2 + 2 - 3 \times 10 \div 2 \times 9 \div 3$ is -1.
   (c) $1 - 2 + 3 \div 4 \times 5 + 6 - 7$ is $-14\frac{3}{20}$ and $2 + 2 - 3 \times 10 \div 2 \times 9 \div 3$ is $3\frac{24}{54}$.
   (d) $1 - 2 + 3 \div 4 \times 5 + 6 - 7$ is 2 and $2 + 2 - 3 \times 10 \div 2 \times 9 \div 3$ is 15.
   (e) $1 - 2 + 3 \div 4 \times 5 + 6 - 7$ is $-\frac{4}{16}$ and $2 + 2 - 3 \times 10 \div 2 \times 9 \div 3$ is $\frac{10}{54}$.