Math 475 Homework 2

DUE FEBRUARY 4, 2010, 11:59PM

1. Prove $3 + 2 = 5$ starting from axioms and definitions from class.

2. State precisely what is wrong with this proof.

   Theorem. *All math majors have the same mother.*

   Proof. Consider the proposition $P(n)$, "For any collection of n math majors, the majors will all have the same mother". We will prove $P(n)$ for $n \geq 1$ by induction on n, the number of students in the class.

   Base Case, $P(1)$. If there is one major, then the collection trivially has the same mother.

   Inductive Step, $P(n) \implies P(n+1)$. Assume for any collection of n math majors, they all have the same mother. Now consider a collection of $n+1$ math majors. Let $p = n+1$. Since n is a variable, we can let $n = p$. By the induction hypothesis, any collection of p math majors have the same mother. Since $p = n+1$, we proved the statement for $n+1$, so we have proven the inductive step. □

3. State precisely what is wrong with this proof.

   Theorem. *All math majors have the same father.*

   Proof. Consider the proposition $P(n)$, "For any collection of n math majors, the majors will all have the same father". We will prove $P(n)$ for $n \geq 1$ by induction on n, the number of students in the class.

   Base Case, $P(1)$. If there is one major, then the collection trivially has the same father.

   Inductive Step, $P(n) \implies P(n+1)$. Assume for any collection of n math majors, they all have the same father. Now consider a collection of $n+1$ math majors, $\{M_1, M_2, M_3, \ldots, M_{n+1}\}$. The last one, M_{n+1} volunteers to leave the room. By the inductive hypothesis, $\{M_1, M_2, M_3, \ldots, M_n\}$ have the same father F. Now M_{n+1} returns and M_1 leaves. By the inductive hypothesis, $\{M_2, M_3, \ldots, M_{n+1}\}$ all have the same father, and since M_2's father is F, then M_{n+1}'s father is F . Therefore they all have the same fathers, and we have proven the proposition for collections of $n+1$ majors.

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