Math 475 Homework 5

Due March 5th, 2010

1. Use the "pattern" method to justify the following:

   (a) Any nonzero natural number raised to the zeroth power equals 1.

   (b) Subtracting a negative number is the same as adding its opposite.

   (c) Multiplying a negative number and a negative number results in a positive number.

2. Tell me which of the integer models is your favorite for describing subtracting and multiplying negative numbers. Use one such model to justify why (-2)(-3)=6.

Use only axioms, definitions and facts proved in class to prove the rest of the problems. (Hint: there is no induction necessary.)

3. Recall that we defined integer addition as $[(a,b)] + [(c,d)] = [(a+c, b+d)]$, for all $a,b,c,d \in \mathbb{N}$. Prove that it is well-defined, meaning we can compute the addition using any representatives of the equivalence classes and get equivalent answers. Specifically, show if $(a,b) \sim (a',b')$ and $(c,d) \sim (c',d')$ then $(a+c, b+d) \sim (a'+c', b'+d')$.

   Integer Notation. For all $a \in \mathbb{N}$, we will abbreviate the equivalence class $[(a,0)]$ as simply a. Except when $a = 0$, we call integers of this form *positive integers* and write the set of positive integers as $\mathbb{Z}^+$. For all nonzero $a \in \mathbb{N}$, we will abbreviate the equivalence class $[(0,a)]$ as ${}^-a$ and call integers of this form *negative integers*. We write the set of negative integers as $\mathbb{Z}^-$.

4. Prove every integer is in $\mathbb{Z}^+$, $\mathbb{Z}^-$ or $\{0\}$. (You may assume trichotomy for $\mathbb{N}$, that is, for all $a, b \in \mathbb{N}$, exactly one of the following are true: $a < b$, $a > b$, or $a = b$.)

   Definition of Integer Subtraction. For $x, y \in \mathbb{Z}$, we write $x - y = z$ iff $x = z + y$.

5. Verify using the definition of integer subtraction that $3 - 2 = 1$, $2 - 3 = {}^-1$, and $2 - ({}^-3) = 5$.

   Additive Opposite Notation. For any $x \in \mathbb{Z}$, we write its unique additive inverse as ${}^-x$. (Previously we only defined ${}^-a$ for integers $[(0,a)]$.)

6. Prove that $x + ({}^-y) = x - y$ and $x - ({}^-y) = x + y$, for any $x, y \in \mathbb{Z}$.

   Definition of Integer Multiplication. For all $a,b,c,d \in \mathbb{N}$, $[(a,b)][(c,d)] = [(ac+bd, ad+bc)]$.

7. Compute 2×3, $2 \times ({}^-3)$ and $({}^-2) \times ({}^-3)$.

8. Prove that for every negative integer ${}^-a$, ${}^-a = ({}^-1)(a)$.

9. Prove that the product of two negative integers is a positive integer.