**Math 475 Homework 3**
Due February 19th, 2010

1. **California Standards**. Acquire a PDF of the California Math Standards. (Googling "california math standards" will get it.) I suggest you not print it out - it is very large. Skim the standards for 6th grade through high school. Imagine that you will be teaching these topics next year. Write a short list of topics that you would like a refresher on. (I'll try to cover the most popular requests this year.)

2. Using Peano's Axioms, definitions and facts proved in class,

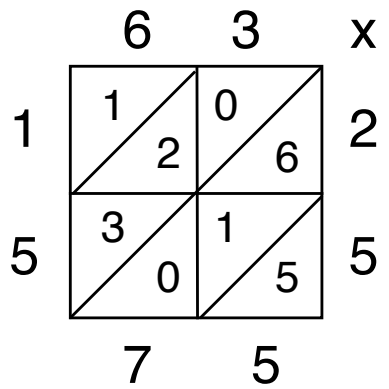
   (a) prove multiplication is right-distributive over addition. That is, for $a, b, c \in \mathbb{N}, (a+b)c = ac + bc$.

   (b) prove multiplication is commutative. (*Hint*: see previous problem.)

3. Using the symbolic method from class where we broke each number into tens and ones and justified each step using the laws of arithmetic, explain:

   (a) why the American Subtraction algorithm (borrowing) works of $64 - 27$,

   (b) why the European subtraction algorithm (adding one below) works on $64 - 27$, and




   (c) why the lattice multiplication of $63 \times 25$ works. You are allowed to assume we know how to multiply single-digit numbers. Your justification should end with the line

$$= (1)(1000) + (2+3)(100) + (6+1)(10) + (5)(1)$$

4. **French Hand Multiplication**. In Butterworth's *What Counts*, p.205, he writes:

   > To this day [about 1930], the peasant of central France (Auvergne) uses a curious method for multiplying numbers above 5. If he wished to multiply $9 \times 8$ he bends down 4 fingers on his left hand (4 being the excess of 9 over 5) and 3 fingers on his right hand ($8-5=3$). Then the number of bent-down fingers gives him the tens of the result ($4+3=7$) while the product of the unbent fingers gives him the units ($1 \times 2 = 2$).

   Explain using algebra why this method works for multiplying any two numbers between 6 and 9. Checking all the cases is admirable, but is not sufficient.

5. **Doubling/Halving Multiplication**. This is a demonstration of multiplying $25 \times 63$ using two columns. In the first column we'll halve the number above (discarding remainder), and in the second column, we'll double the number above. We'll stop when we get to 1.

| | |
|---|---|
| 25 | 63 |
| 12 | 126 |
| 6 | 252 |
| 3 | 504 |
| 1 | 1008 |

Now we mark every row whose first entry is **odd**.

| | |
|---|---|
| 25* | 63* |
| 12 | 126 |
| 6 | 252 |
| 3* | 504* |
| 1* | 1008* |

Now add all the second entries of the "odd" columns. So the answer is $63+504+1008 = 1575$. You might think this is a silly way to multiply, but notice that this method only requires recognizing odd numbers, adding numbers and figuring out halves. Also, it does not rely on place value! This makes it a relatively usable method for non-place-value number systems, like Roman numerals. (Rhetorical question: How else could you multiply with Roman numerals?)

- Actual question: Explain why this multiplication procedure gives a correct answer for any two numbers.