Changing Teachers' Conception of Mathematics

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Introduction

This paper describes the efforts of the authors to work with a group of in-service and pre-service math teachers to change their conception of mathematics. We describe the teachers' starting conception and our desired re-conception. We then detail our efforts to craft moving experiences that would shift their thinking. We discuss why it is insufficient to "just tell them" and conclude with a cautionary note about the differing experiences of our teacher leaders who did or did not craft similar experiences for their fellow teachers.

WHAT IS THE ESSENTIAL WORK OF MATHEMATICS?

Mathematicians are often asked, "What do you do exactly? Haven't all the math problems been worked out?" Many people seem to think this is a very reasonable question, because what they know is from a traditional school mathematics experience, organized to guide them to memorize definitions and recall known routines to solve known problems.

To compare this common view of mathematics with the popular idea of what a scientist does, imagine a person in a lab coat tackling an unsolved challenge, performing experiments (some of which fail and explode, some of which finally succeed), getting stuck and frustrated until finding an alternate approach that causes a great "Eureka!" or perhaps making an unexpected connection to other knowledge, and then arguing with skeptical peers about results and plans and finally convincing people with logic and passion. This popular perception, perhaps glamorized, is essentially accurate, and the spirit of this description fits the work of mathematicians as well (perhaps without the lab coats and explosions).

Re-stated, the essential work of science and mathematics is to:

- Analyze non-routine situations;
- Explore a situation, fail productively, and persevere through frustration;
- Find new and multiple ways to approach a problem;
- Connect knowledge and relate ideas to form new understanding; and
- Communicate ideas and use logic to convince others of their value.

It is unnatural to educate students mathematically without their experiencing this essence. Yet the traditional school math experience emphasizes mental discipline and mathematical literacy, which are indeed important, but at the cost of losing the whole substance of mathematical thinking.

Furthermore, there are practical ramifications of this traditional approach. First, the more disconnected the routines are, the more difficult it is to remember them (NRC 2001, 2005). Second, the boredom of barren tasks drives interested minds out of science and math. Third, such a presentation filters strongly for students who have an existing faith that memorizing and reciting is important to their future success (NRC, 2001, 2004, 2005).

WORKING WITH TEACHERS LOOKING FOR CHANGE

Usually, math teachers have themselves been educated in a system where math is taught and valued in bits and pieces. They then perpetuate the vicious cycle when they organize their students' math experience in the same impoverished way. The authors had an opportunity to try to break the cycle by working with teachers who were motivated to change their teaching.

REvitalizing ALgebra (REAL) was an NSF Math Science Partnership aimed at improving the teaching and learning of algebra by developing teacher leaders and sparking lasting change in math departments at secondary schools and universities. The teachers in the project came from three different teaching settings: secondary mathematics teachers, graduate students in mathematics at San Francisco State University (SFSU), and undergraduate mathematics majors at SFSU.

For REAL, we selected teachers who said they believed changing their own teaching would enable their students to be more successful despite outside influences on the students' lives or constraints of their schools. The secondary teachers were teaching algebra, the undergraduates were paid to assist in the secondary classrooms, and the graduate students were teaching remedial algebra courses at SFSU. We worked with two cohorts of 27 members each consisting of approximately nine secondary teachers, nine graduate students, and nine undergraduates. The three co-directors worked with each cohort in a three-hour class each week through the first academic year and then everyday for three weeks during the summer. During the second year, supported by reduced teaching loads, secondary teachers led teacher teams in their home departments and math graduate students were paid a stipend to work with colleagues who were teaching in the mathematics department. (There was no follow-up for the undergraduates, who had been paid to support the secondary teachers, beyond encouraging those who were interested to enter teacher preparation programs.) We refer to participants from all three groups as 'teachers.'

During the semester prior to beginning work with the teachers, the three co-directors spent a half-day a week observing their classes. Their observations were not structured but they observed the lessons and the reactions of the students from the viewpoint of mathematicians and experienced mathematics teachers. At the same time the evaluator, Katherine Ramage, used a framework for observations developed jointly with the co-directors and attached to this article as Appendix B. Although we did not use a formally validated observation tool, the observers for the evaluation who used this tool regularly conducted inter-rater reliability checks, and these results showed good reliability.

From our extensive classroom observations and initial conversations with our participants, we found that, for the most part, teachers saw math as a batch of rules and facts, or at best, an ordered list of isolated definitions and procedures to be taught by them and remembered by their students. Textbooks and standardized tests determined the content and sequence of the math they taught. Teachers reported experiencing their own learning of mathematics as having problems presented that they would then solve by searching their memories for a statement or procedure that, when applied, would give them the answer. They also remembered respecting those who could show what they knew by solving problems quickly, often in a matter of minutes.

Creating A New Conceptualizing of Mathematics

Inspiring sympathetic teachers to enrich their conception of mathematics was the challenge. The heart of our approach was to create situations where they could:

1. Enjoy rich mathematics as students;
2. Practice identifying rich activities;
3. Practice facilitating such activities as teachers; and
4. Work towards believing their own students could learn in this way.

We also spent time in the workshops dissecting math problems and explicitly detailing connections among different parts of the algebra curriculum.

After we describe these approaches, we will reflect on why we took the trouble to set them up. Charisma and authority are not enough to change deeply ingrained beliefs. Logic was not enough to convince. We had learned these lessons before, but we re-learned them during this project, and probably will re-learn them in the next.

TEACHERS ENJOY RICH MATH AS LEARNERS

For teachers to change their ideas about the nature of mathematics, they first needed to experience 'doing mathematics' as we envisioned the subject. During a part of every REAL meeting, both during the academic year and the summer program, the teachers worked in groups on rich mathematical problems. We define a rich problem as having the following attributes: (Hsu, 2007)

- The "mysterious" part of the problem is mathematical.
- The problem has very little overt scaffolding.
- There are several ways to do the problem.

- Students of different skill levels can learn from this activity.
- The problem has natural extensions.

To solve these problems, teachers had to explore. They could not immediately see a procedure or a theorem to apply. There were usually several ways to begin exploring. In every problem some ways of exploration were accessible to the teachers with the least formal mathematical training, and those teachers often had insights that were of use to the most sophisticated members of the group. We randomly assigned the teachers to problem solving groups, so for the most part, there would be a mix of levels of mathematical sophistication in each group.

As a result of these tasks:

a. Teachers experimented and failed productively.
By working on these problems, the teachers saw that solving a mathematical problem involved "getting their hands dirty" through exploration and not sitting back to wait for a bright idea to come to mind. They began to see trial and error and learning from mistakes as legitimate and necessary mathematical problem solving tools that could be used as tools for learning.

For example, looking at a problem such as "Which numbers can be written as the difference of squares?" teachers would start looking at differences of squares such as $4^2 - 3^2 = 16 - 9 = 7$. After looking at some more examples, they might conjecture that all differences in squares are odd numbers. That they could not write 2 as a difference of squares was a verification, but then someone tried $4^2 - 2^2 = 16 - 4 = 12$, and they would see that their initial conjecture was wrong and needed modification. The teachers learned that being wrong was not detrimental; it helped them think toward a solution. In mathematics, genuine problem solving will proceed in a fitful manner. It will not normally proceed smoothly. Teachers needed to realize that when they protect their students from being wrong or thinking incorrect thoughts, they are keeping their students from solving problems on their own.

b. Teachers saw that people weren't just "better" or "worse" at mathematics.
On the occasions when our random process turned up homogeneous groups, the least sophisticated groups sometimes took the problem farther than those who had taken more direct mathematical approaches. Teachers learned that they could not assume in general that one person is 'better' than another mathematically as they saw different colleagues excel on different tasks and they began to see mathematical talent and learning as a mix of attributes.

Teachers commented on getting better at things where they were initially weak, such as visualizing. They learned by doing and by working with others who had different strengths and commented in discussion and in their daily written comments on using strategies they had not used before.

We wanted these experiences to keep them from pigeonholing their students as starkly as they had done initially when they talked about "strong and weak" students or "low students" and about their classes as "low level" or "slow algebra."

c. They worked together and learned from each other.
The problems were chosen so there were a number of non-routine insights needed. This was meant to make them explore as in (a), and to break down status differences and stereotypes as in (b), but also so that they would need each other's help! We wanted them to see the value of having students work together, and we wanted them to believe that their students could learn in this way, and that it would not be 'weak' students learning from 'strong' students with the 'strong' students being burdened by teaching and not learning anything new.

In particular, a non-routine problem makes people argue about the mathematics and about problem-solving strategies. An essential piece of the work was putting our teachers in situations where they had to communicate using mathematics. Many of our teachers were not used to communicating about math to investigate, to question, or to convince. They were used to being the classic sage on the stage transmitting well-honed signals to their students.

d. Teachers enjoyed doing mathematics.
Almost all teachers agreed that their favorite part of the REAL class was working on the problems. Many admitted they had not really enjoyed doing mathematics in the past. In the past, they may have been good at getting the answers and found joy in recognition that comes with success or in the approval their teachers, but in their anonymous evaluations they claimed the joy of actually solving a problem and knowing that they'd done it was greater. This piece should not be undervalued. It is amazing how many students

come into a math major loving to do mathematics and leave it, perhaps with respect for their hard work at mastering difficulties, but without actually having that same sheer enjoyment of the mathematical work. Others, who were successful in high school, never reach the point of enjoying mathematics in college.

TEACHERS DEVELOP TASTE IN SELECTING PROBLEMS

Immersing teachers in rich problems helped them develop skills for solving them and a taste for working on them. However, many teachers did not have the ability to recognize a rich problem when they saw one. We had experienced this lack of recognition in previous teacher education projects so we were prepared to work on this issue. The interesting thing is that the language is slippery enough that teachers would agree that the five aspects of a rich activity were desirable, but these teachers would then have trouble using that language to identify productive problems. Part of this uncertainty comes from little experience working on rich problems, and part of it comes from a lack of experience in facilitating student work on rich mathematics. There is an art to seeing the possibilities in a problem and to seeing how possible solution paths can lead to interesting mathematical discussions. In general "taste" is hard to describe but becomes shared through repeated experiences. It became a term we used internally to describe teachers' evaluation of problems in terms of their richness.

We weren't sure how to cultivate this taste, so we took the simple approach of finding a number of tasks that we ourselves thought were rich and ones that were not so rich and then gave the mixed list to groups of teachers to sort. We followed the sorting with a whole group discussion of each problem.

In one activity, we asked teachers to work with a partner to search for two problems on the web – one that was rich and "group worthy" and one was apparently rich but not really. We asked them not to label them and write each on an index card. We then selected six from those submitted and wrote them on the sheet shown in Figure 1.

Teachers worked in groups of four to sort the problems into rich and not rich. Then we had a whole class discussion about which problems were rich and how ones that were not might be made richer or used for another worthy purpose (Hsu, 2007).

We also engaged in some important complementary activities where we asked participants to try to enrich problems by removing some of the overt structure or specific directions for tasks. (This is described in detail in Hsu et al., 2009.)

As a result of these tasks:

a. Teachers practiced analyzing a problem in detail and its value for inspiring mathematical thinking.

That is, a problem isn't about a subchapter in a textbook where it appears, or about the kind of problem it is and the recipe that solves it. A problem is a task that inspires student thinking and enables them to develop mathematical solution methods in a teacher facilitated group work setting. The teachers began discussing what mathematics could be brought out in approaches to a problem and what kinds of connections could be made by sharing multiple solutions. From this point of view, many of the mathematical tasks we give in classrooms are impoverished, often meant to inspire a single mathematical approach. Sometimes these kinds of problems are necessary, but it is important for us to know what we are sacrificing.

There are no set rules for when to direct students and when to let them explore. A certain amount of material needs to be covered, thus a certain amount of direction is needed. At the same time, a certain amount of richness needs to be present to make for an engaging class. Students need time to explore on their own, but teachers are constantly faced with demands to focus on what appears to be the content of the tests. How much of each kind of instruction to include is a judgment call and making that call is part of the art of teaching. It all depends on the teacher, the curriculum, and the students.

b. Teachers began adapting their tastes while productively saving face.

Some teachers would initially be satisfied with the level of richness of a rather limited problem. But through our pushing of the discussion and through listening to their peers, people began to raise their expectations for what was a rich problem. They were usually able to save face by noting that the problem would be rich for students who were sufficiently inexperienced. Indeed, this observation can be made of most tasks—even mundane mathematical recipes are intriguing to those who have never been taught the recipe. We thought this face saving was in itself a worthwhile lesson.

FIGURE 1

How Rich Are These Six Problems?

1. Within Eldoria, a little country far away, you can place a call with one of two companies.

   - *EZ phone charges $24/month for the first 3 hours and then 8¢/minute.*
   - *U-Call phone charges $30/month for the first 2 hours and then 5¢/minute.*

   a. After how many minutes of local calls will the two plans cost the same?

   b. Make a graph of each cell plan on the same set of axes. Make sure to label your axes.

2. There are many rules that fit the information in the In | Out table below:

In	Out
5	16

   a. Your task is to find at least 10 different rules that work. You can use multiplication, division, addition, subtraction and exponents and you can use more than one operation in a single rule.

   b. The table below has a bit more information than the one above, but that only makes things more interesting. Find as many rules as you can that fit both rows of this table.

In	Out
1	2
2	5

3. a. On the same set of axes, plot the graphs of $1/2\ x^2$, x^2, $2x^2$.

   b. On a second set of axes, plot the graphs of $-2/3\ x^2$, $-2x^2$ and $-x^2$.

   c. Write a paragraph explaining the ‘a’ in ax^2 affects the graph of x^2.

4. Two of the most commonly misused laws are called “the product of powers” and “the power of a power.” Aka: $x^a \bullet x^b = x^{a+b}$ and $(x^a)^b = x^{ab}$, respectively.

   Task: Prepare an explanation of these laws, as if teaching someone who is learning this for the first time.

5. Take any three consecutive integers. Square the middle number and multiply the first and the third numbers. Compare your answers. Use algebra to find out why this will always happen.

6.

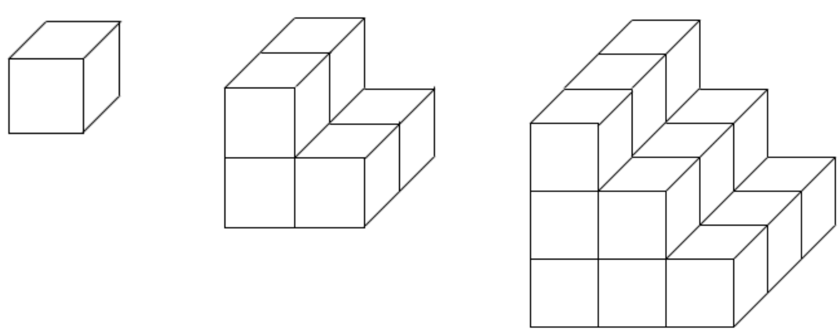

Figure 1 Figure 2 Figure 3

Each figure is constructed of cubes (1 cm x 1 cm x 1 cm).

   a. Find the surface area of figures 1, 2 and 3.

   b. Find the surface area of the 50th figure.

Though we do not have an objective measure of "taste," we do believe that most of our teachers improved their taste over time. Comparing their proposed rich tasks from early in the workshop to later ones, it did seem that the richness of the tasks increased, although we were impatient with the rate of change.

TEACHERS LED PROBLEM SOLVING IN SMALL GROUPS OF GOOD WILL

The tasks described so far were intended to give teachers the opportunity to enjoy active learning and develop the taste to select rich activities. But there was still a gap between these experiences and their confidence that they could create and sustain such learning situations in their own classrooms. It is a subtle and difficult task to support teachers in facilitating group work among their students and we probably did not budget enough time for this in the workshops.

In REAL, our strategy was to first deal with the difficulties inherent in facilitating group work even in an ideal situation of student goodwill and then introduce other tasks to handle teacher difficulties in non-ideal situations.

We wanted to create a situation where people could concentrate on facilitating group learning away from the stress of difficult students, unrealistic academic calendars, and unhelpful curriculum materials. The problems were chosen so as not to require any mathematics beyond algebra, partly to emphasize the richness of the mathematics in algebra, and partly to avoid giving advantages to those teachers who were concurrently studying advanced mathematics.

First we facilitated their work in groups to solve selected problems so they would experience the problem they would be teaching as a student. Then they worked together to prepare to teach the problem to a subset of the class. Nine teachers co-taught in trios. Each trio taught two of the other problem groups, so they had a class of six students, which typically would be run as either three pairs or two trios of students. Learners gave anonymous feedback on index cards to the teachers, and afterwards, the problem teams reconvened to compare notes on their experiences either as teachers or learners. There were no directions given to the learners regarding their comments; only that they should make comments they thought would be useful to the teachers.

In the first session we allowed freedom in choice, which resulted in a wide range of teaching approaches, some more engaging than others. In two subsequent sessions, we gave more explicit guidance to focus on certain aspects of teaching, such as asking good questions, handling unequal participation within a group, or how to move groups to key checkpoints in the activity without ruining their creative thinking. We began the guidance through a discussion with all the teachers, getting them to specify problem areas for their practice teaching. Then we asked them to prepare in their working trios strategies for addressing these problem areas. One of the co-directors met with each trio to discuss their strategies. Finally, after the teaching episode, the trio reflected on the success of their plans.

As a result:

a. Teachers found it was really fun to facilitate a rich problem with interested students.

It's a great feeling to manage an experience for people who get excited and engaged. This is a feeling that our teachers did not typically get in their classrooms. Even if the situation we set up was artificial, it was a real reminder of the potential joy that comes from helping interested learners.

b. Many teachers were surprised to see people could struggle and solve problems.

Because we did not initially force the facilitators to let people struggle, teachers used a wide range of approaches, from letting people struggle with encouragement and well-placed questions and hints to telling people which path to take and then explaining the answer at the end. Initially leaving the choice of facilitation methods to the teachers provided the basis for insightful discussion when their 'students' shared their reactions.

Teachers saw that intervening lightly by giving learners more time to struggle, offering fewer directive hints, and asking them to describe or explain their thinking often gave people a chance to persevere and to come up with marvelous insights on their own without being told.

c. Many teachers were surprised when their helpful interventions and explanations were not welcomed.

Some teachers were surprised to see people struggle on problems that they, as the facilitators, perceived as simple, but most remembered how they themselves had not immediately found productive paths on their first

FIGURE 2

HW 17 (2/8/05)

1. (ALL) Please discuss one class comment from the Flag Hoist discussion that you agreed with and one you disagreed with.

2. (Teachers & Grads) Think about an algebra problem that could provoke an interesting class conversation with your students, as in the 'Flag Hoist' video. Plan to give your class the problem during class or for homework and have a 'good' class discussion about the problem before Feb 22 (in two weeks). At the end of the discussion, give your students an anonymous quiz on the content of the discussion. Bring them to class on Feb 22. (It should be a VERY short, one question quiz.)

3. (ALL) Reading. Read (or reread) the "Messy Monk" article. Imagine yourself in the author's role. What do you think would be the most difficult point in the lesson for you?

Copes, L. (2000). Messy Monk Mathematics: An NCTM Standards-Inspired Class. *Mathematics Teacher*, 93(4), 292–298.

HW 18, Due 2-22-05.

1. Remember that at the end of the rich discussion you gave your students an anonymous quiz on the content of the discussion. Bring the quizes to class on Feb 22. (It should be a VERY short, one question quiz.)

2. Read *The Nature of Classroom Teaching*, Ch. 2. On pages 2 and 3 from the article the author gives an example of a situation that is mathematically problematical for students and one that is not mathematically problematic. Come up with one example of each from algebra.

3. Read *Engaging Schools*, Chapter 1.

4. Respond to the following questions in relation to the *Engaging Schools* reading.
   - How can you tell whether a student is engaged in your class?
   - What strategy did a teacher use in a class you took that got you engaged?

encounter with the problem. In fact, a substantial proportion expected people not to be able to solve the problems without their hints to push them along to the teacher's solution.

Heavy-handed intervention and explication by facilitators was often met with resentment for "stealing the thunder." Facilitators were sometimes surprised when their 'students' liked their own solutions and representations better. Needless to say, it was unusual for teachers to get such honest and constructive feedback in their normal practice.

d. Co-teaching made visible the many choices and mathematical observations a teacher makes.
Each co-teaching trio found themselves discussing where the class was and how to choose the next move at each step. Occasionally one of the trio would act as a silent observer, but in general, the trios naturally discussed key teaching moves, such as whether groups were working quickly enough, which groups needed help with math or with internal status differences, which groups ought to present and in which order. Even though we did not require them to consult with each other, it naturally occurred in all cases.

TEACHERS STROVE TO BELIEVE IN THEIR STUDENTS

In the previous tasks, teachers grappled with the subtleties of handling group work and rich problems with very cooperative students. But for teachers to incorporate this new sense of mathematics in their own classrooms, they needed to become convinced that their own students were capable of doing mathematics in this way, and that their students could learn important content in this way.

Many said their advanced students would work on problems in ways similar to those of their colleagues but they were not convinced that the students who had trouble with algebra were capable or would be willing to work in this way.

To convince teachers that all of their students could benefit from working on rich problems, we gave them assignments that involved teaching their own classes in new ways and then reflecting on what happened. See Figure 2 for two consecutive assignments. In our observations of the secondary math teachers, we saw very little change during that first year. Based on our records of classroom observations, it was not until the following year that change was observable. It is unclear whether teachers needed time to integrate the new approaches or if they needed to start fresh with new students to make such big adjustments. These secondary math teachers also faced directives and mandates from their districts around curriculum materials, especially for students failing algebra, which may have contributed to their pace of change. The graduate instructors did incorporate changes in their initial year, but their classes were only one semester long, so they started with new students in the middle of the year. The graduate students were also relatively new to teaching algebra so may have found it easier to try something different.

In addition to asking the teachers to try new lessons, we showed them tapes of some lessons, which are described in Appendix C. Some of the classrooms in the tapes were approaching mathematics in the new way and others were of more traditional classrooms. The teachers were not given specific aspects to watch for, even though we always had a list of topics to discuss. The facilitator would ask for comments on positive aspects of the instruction and then open the discussion for general comments. At first teachers were reluctant to criticize other teachers, but later in the program, some became overly critical. In either case, starting with positive comments served to balance and enhance the discussions. The differences in the degree of engagement of the students in the tapes where instruction was student-centered compared to those that were teacher-centered were remarkable, and some of the student-centered tapes were of classrooms with students from lower socio-economic levels. It was, however, very difficult to find good examples of student-centered classrooms in urban schools. This lack of evidence made it difficult to convince some teachers that their students would be capable of learning in such a classroom.

We assigned readings about lack of success in mathematics learning for African-American and Latino students. See Appendix A for the list of readings we used. These articles were discussed during the REAL class, and during the follow-up year, the graduate students and many of the secondary teachers, who had participated in the first year, chose to read and discuss these and other articles in their meetings with their department colleagues. But some still remained unconvinced that their students would have the ability to learn mathematics in a student-centered environment.

Our first task in attempting to convince teachers that the important content could be learned through problem-based lessons was to give them the vision of what such curriculum would look like. The mathematics problems the teachers worked on in the REAL class, were usually not designed to teach particular content since we wanted the teachers to work together as equals on the problems; although, their backgrounds with respect to mathematics content varied widely from very few college math courses to graduate students working toward a masters degree in mathematics. Together we examined problem-based lessons from reform curricula, which approached algebra in a student-centered way.

The teachers needed to realize that all the mathematics they had been teaching as separate procedures could be looked on in a different way. The lessons from the reform programs provided an opportunity to see their curriculum organized around big ideas. In Hsu et al. (2007) we reported on our experience working with teachers to identify and build activities around big ideas. However, pressure for students to do well on state tests was a major impediment to using a problem-based approach. Through readings and discussions, we succeeded in convincing teachers that tests composed of many problems that required rapid application of procedures did not assess whether students could reason and solve problems nor their understanding of concepts. However, most believed that their mathematics programs would be in jeopardy if their students did not perform well on those tests. We needed to convince them that their students could do as well or nearly as well if they did not teach to the test. This turned out to be an extremely difficult task, and most decided to compromise and teach in a student-centered way part of the time, but to teach to the test some of the time also. Unfortunately, there is some evidence that mixing these approaches may not work (Pesek, 2000).

Once teachers had a vision and believed they should change their teaching, they were still not confident that they could conduct class in this manner. They feared behavior problems, that in groups only the 'fast' students would do any work, that many groups would give up, and that they, as facilitators, would not be able to get them restarted. There is much to learn about group facilitation and good facilitation is vital to running a student-centered classroom. Through our work in REAL we realize that this third concern requires continuing, follow-up support, far more than our project was able to offer. We discuss some of the issues of helping teachers to acquire these skills in an article on differentiating instruction (Hsu et al., 2007).

Why Bother With These Moving Experiences?

Some of the participants were frustrated by our refusal to "just tell them clearly what we wanted them to do in their classrooms." In fact, much of this frustration was demonstrated by teachers who eventually came to strongly respect our judgment through some mix of their beliefs, our authority, and their experiences. The transformation seemed to occur over the summer when they were away from us and had more time to think. They had fought the new ideas we were supporting during our classes, but in the fall, they began to practice them with fervor in their own classrooms. When we visited their schools to observe, they had rearranged their classes to accommodate groups, they were assigning fewer but meatier problems for class and for homework, and they were structuring their teaching around big ideas as opposed to isolated procedures.

Some of the frustrated participants came to believe we used our methods out of our own ideological commitment to constructivism, and that we wanted them to invent the answers themselves. But this was only a small piece of the puzzle. If we could just tell them what to do and have them go forth to be great teachers, we would. But we believe they really needed the moving experiences and new images of how students learn to understand to the level of turning words into practice.

WHY NOT JUST TELL THEM WITH CHARISMA AND AUTHORITY?

Perhaps the first idea that comes to mind is that if we could explain our conception clearly, repeatedly, and with charisma and authority, the participants would be able to internalize it. Indeed, we did occasionally share our opinions and educational values in the course of our project, especially when a co-leader was in the audience as a fellow teacher. It would have been a very short project if this were sufficient. But cognitive science and research on how people learn (NRC, 1999) make it very clear that relying on charisma and authority alone has serious limitations.

Everyone has been inspired by hearing a speaker. But most people have also had a follow-up experience of not remembering the details and logic, or even worse, losing interest upon reflecting soberly, and perhaps feeling fooled. This is like the math students following a wonderful speaker and nodding along and then afterwards realizing they didn't really understand. Similarly, for instance, we had a healthy, enjoyable discussion in the first weeks of our workshop where most people agreed that it was important for us to have student activities that engaged the students and had them think about the concepts underlying rules and definitions. Then we sent them away to put together a sample conceptual lesson. When we saw the results, we were sobered. Only two of the twenty-seven activities they produced met our criteria for a conceptual activity, and one of those was by an undergraduate who wasn't yet teaching her own class! This experience was to be expected, because these ideas are subtle and hard, but even though we expected that it would take time the results were an unpleasant surprise.

Even if we delivered our message with such charisma that everyone would be inspired in a lasting way, we could not guarantee that they would do the same as lead teachers. In our project, we wanted change to spread from the lead teachers we worked with directly to their department colleagues in secondary school and the university through the meetings in the second year. Even if they accepted our authority, as many of them did, none of them held the same authority with their peers.

WHY NOT JUST LOGICALLY CONVINCE THEM?

A second idea might be to not rely on authority and charisma, but to simply state the change we wanted to see and make a strong logical argument. Then we could allow teachers to debate the issues and the logic would convince.

1. These words are easy to misunderstand.

Some words are not easy to misunderstand, like "abelian" or "polynomial," because they can be defined rigorously. However, in educational work we can only use approximate words with as many different meanings as there are people. For instance, take merely one brief phrase used in the opening, "analyze non-routine situations." What does it mean to analyze? Some teachers thought it was enough to provide a numerical answer with some related computations. Some wanted an argument, but only a rigorous one. Some welcomed partial answers and creative approaches even if they were not well articulated.

Then, what is a "non-routine" situation? In the course of our work, we found some teachers would count a standard problem type whose numerical values were hard fractions as non-routine. Others took a routine problem and appended a fun non-mathematical task, such as using the solution to uncover a secret phrase or answer a riddle. Others posed a question that did not have a known recipe for solving it. Even the word "situation" is trouble. Are "situations" always realistic world settings? Can they be fanciful? Can they be abstract situations?

If we could precisely define what we mean by "analyze non-routine situations," we would have done so. But there was no way to do this in words. The best we could do was to provide shared experiences and to ask the participants to discuss them. While this practice led to some frustration among the participants, there does not appear to be a simple way of getting the notions across.

Multiply this ambiguity across the whole range of vocabulary, and it is almost impossible to have genuine discussions about these issues without actually experiencing the exploration, analysis, and communication together. These common experiences are necessary to create meaningful common vocabulary.

2. These words are loaded.

Some words sound so good that everyone aspires to those labels. Everyone wants to believe that their students are "engaged" and understand things "conceptually" and that they are teaching "the big ideas" of the course. But we believed many of our teachers had never experienced the depth of engagement, conceptual thinking, and course conceptualizing that we pushed for, so simply agreeing to value those would be pointless.

Also, in the wake of the math wars, many terms were loaded, and people who considered themselves on one side or the other of reform or tradition had prejudices towards "group work" or "basic skills" and other hot-button phrases. These prejudices interfered with real conversation about what effective group work or practice looks like. More subtly, there was a small but important core of teachers who insisted they were above the whole reform-traditional debates and took the best of everything in moderation. These teachers seemed the least willing to change their practice, as if any suggestions to them would perturb their equilibrium, and later they turned out to be the most upset that we were not directly prescribing a position. They suggested that our putting them in rich learning situations was a way of tricking them into agreeing with reform beliefs.

3. We can't give them recipes for the whole wide range of future challenges.

Mathematics is more than a collection of problem recipes. Teaching mathematics is more than a collection of teaching recipes. A number of our teachers admitted that they had hoped we would set out for them which excellent teaching to do and which great problems to use and then pinpoint when to use them.

A Test: Teacher Leaders Try To Move Department Colleagues

We conclude with a striking example of the importance of colleagues working together on rich mathematics problems, namely the experiences of project secondary teachers working with their departments. Our secondary teachers spent the second year of their project participation as lead teachers for teams of their math teacher colleagues (whom we will refer to as "department teachers"). The secondary school teachers, both the teacher-leaders and their department colleagues, were given a released period for a full year to participate and the graduate instructors were paid stipends for their time. All the groups met multiple times each week.

We had decided to let the lead teachers determine the form and the pacing of the teacher meetings. We visited their team meetings on a weekly basis along with many of their classes, and we gave feedback as to what we saw happening. We made suggestions about activities they could do as a department. We continually reminded our lead teachers of the ultimate goals of having authentic conversations about improving practice, getting department-wide commitment to looking honestly at their teaching, and making this part of their department culture. But while we pushed the big ideas and concepts of the professional development, we left the details up to them out of respect for local autonomy.

Response was different at different sites. The eight school teams exhibited different levels of engagement. The first level, which every teacher team reached, was to work on curriculum materials together and to collaborate to insert some isolated special rich activities. Only four secondary math department teams reached a next level, where the department's culture changed so that the teachers became part of a community that worked together on mathematics, on teaching and learning, and on sensitive issues of race, ethnicity, and expectations. In addition to working on curriculum materials they spent their meeting time reflecting on and discussing tough issues, and made decisions with effects that were apparent in their day-to-day work in the classroom. These four departments were the ones that started their meetings together with work on rich mathematics problems and then worked through a sequence of activities similar to the ones they had experienced. One other team had already developed a department culture that supported their work on improving methods of instruction but they did not really progress beyond where they had started.

In retrospect, we believe the lack of stronger guidance during the second year was a mistake. Most of the lead teachers did not have the intellectual leadership skills or enough pre-existing status in their departments to facilitate activities that involved risk taking (Hsu, 2009). The schools that did have strong lead teachers used a number of the approaches, which we have described, and they did move their department cultures to change during the project. The other lead teachers did not push their teacher teams through the initially uncomfortable engagement in solving mathematics problems together that appears to have been a necessary step toward change. Because as project leaders, we potentially did have the authority and capital to push the teams to that uncomfortable place, we are left with a big question. Would mandating the use of the approaches described in this paper and working with department leaders to co-facilitate the school-site meetings have caused deeper and more lasting change? Maybe all the teachers needed to participate directly in the whole REAL program.

Conclusion

Most of the teachers in the REAL project expanded their views of mathematics and of what mathematics is important for their students to learn. Those teacher leaders, who took the next step, used what they had learned to facilitate problem solving with their colleagues and lead them into both deeper mathematical discussions of curricular issues and discussions of more sensitive issues of teaching and learning expectations.

It is worth noting that only those departments that started by solving problems together moved on to the other activities: identifying rich activities and working towards believing their own students could learn in this way. And only those teachers made lasting progress toward changing the culture of their departments to include ongoing discussion of mathematics and of improving their teaching in order to support success for students of all racial and ethnic groups. We conclude, as we did in an earlier article on mentoring, that working together on mathematics problems allowed teachers to relax their defenses and start to build the trust needed to participate in frank discussion of more sensitive issues (Hsu, 2009).

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APPENDIX A

REvitalizing ALgebra | ***Readings on Cultural Differences***

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APPENDIX B

REAL CLASSROOM OBSERVATION FRAMEWORK (2005-06)

Creating a classroom culture which/where

- Requires students to respect each other and the class. This might manifest in students being forced to explain and listen to each other.
- Empowers students to speak about personal topics and math. (Look for evidence that it is valuable use of class time to have students speak about personal topics.)
- Students are working on improving cooperation in groups, e.g. silent activity of having to find out the rules without talking.
- Teachers are thinking about groups. Students are used to working in groups and the room is set up for groups.
- Teachers are extending wait time for students to think and then talk.
- Teachers are displaying compassion for students, valuing personal connection as part of classroom atmosphere, e.g., longer wait time, interest in the wrong way to do a problem, making personal contact with student after the student didn't answer.
- Teachers are recognizing incorrect solutions and spending time examining students' thinking.
- Teachers are interested in why students are misbehaving rather than assuming they are bad. Look for evidence that teachers are thinking about what math will make them behave better.
- Teachers are getting around to all students. Classes are not dominated by a small group of students. (Look for evidence of inclusion)

We want to see evidence of teachers

- Giving consideration to quality of activity over coverage, e.g., taking more time for activities that need it, going outside for activities such as "To the Moon."
- Focusing on the big idea and creating a context.
- Focusing less on procedures.
- Moving away from primary focus on correct answers
- Including ELLs, not making it easier for students to read but making them have to cope with the language. Having students read aloud is good as is working in groups. Not depriving ELLs of good math.
- Teacher's thinking moving towards honors and remedial classes looking the same, with both taught conceptually.

When using reform curriculum materials, we do want to see teachers

- Pushing students to explain
- Offering good alternate explanations.
- Improving the quality of math problems making them more challenging.

(cont. on next page)

APPENDIX B *(cont.)*

We don't want to see teachers

- Discouraging students to use concrete supports like manipulatives.
- Skipping the challenging problems and problem solving strategies.
- Making students do all problems.
- Going directly to teaching FOIL rather than letting students discover.

When using traditional texts, we want to see teachers

- Doing something different such as putting the math into a context
- Letting students discover and evolve their own understandings, e.g., laws of polynomials, figuring out proportions on their own rather than the teacher giving cross multiplication.
- Improving the quality of math problems, making them more challenging.

With regard to teacher knowledge, we want to see teachers

- Having a rich understanding of math evidenced in group work supported by rich discussions.
- Thinking about the question "Why are we doing this?" and lingering more on problems and big ideas like slope.
- Realizing that a really "bad day when students struggle and don't reach conclusions" is a good day. They have to come back to it and resolve it later. (Eric thinks that he came up with a bad problem if the students get it right away.)

With regard to assessment, we want to see

- Richer, more embedded assessment.
- Teachers not as frightened of or driven by standardized tests.
- Teachers asking students to solve problems in more than one way and reducing the number of problems.
- More authentic questions and fewer procedural ones.
- Multiple measures, anything that differs from tests and quizzes.
- The ultimate— stopped using class time for test prep.

We don't want to see

- A small number of high stakes assessments.
- All individual quizzes and test.

APPENDIX C

SOME VIDEO MEDIA USED IN REAL

TIMSS 1995: US and Japan. These were once available on videotape, but now can be downloaded at http://timssvideo.com/97 and http://timssvideo.com/67 . Used to contrast questioning of US classroom (short wait time, fill-in-the-blank, quick arithmetic) with the Japanese classroom (problem solving, sense-making, student presentations, computer graphics).

Flag Hoist and Plugged Funnel from Miriam Sherin's VAST Project. Excellent example of two tasks which provoke rich classroom discussions around productive, authentic mathematical disagreement. The tasks also support kinesthetic thinking.

Where is the 10? from Jo Boaler. An example of a rich problem with many ways into it. The video shows some intense group work where the group dynamic and teacher facilitation keeps everyone persevering through a difficult task.

Candle Questions from Driscoll's Fostering Algebraic Thinking. Show parts 1 and 3 to give two contrasting examples of problematic questioning strategies (highly non-directive versus not pushing for explanation) along with challenges of unequal status in groups.

Getting Around to Groups:
- TIMSS 1999, US http://timssvideo.com/58 and
- Sandie Gilliam Group Work Highlights
http://gallery.carnegiefoundation.org/collections/quest/collections/sites/gilliam_sandie/archive.htm *(Constraints videos and Reflections)*
Pair these two videos as an introduction to group work. The TIMSS 99 video shows a teacher inefficiently trying to use direct instruction for each separate group. Sandie has a far more restrained approach. Her class is an IMP 3 class, so they've had three years to socialize into passable group work. She walks around and monitors and intervenes only to push groups along. A highlight is the video "Whole group discussion, Eliminating constraints 2" which has the fascinating piece where students keep working on the problem together over break, including one boy hitting another for not letting a girl participate.

TIMSS 1999 Exponents
http://timssvideo.com/69
An interesting class where a teacher gives students exponent laws and asks them to prove the 0 case and negative numbers case. Students aren't given enough time to work through the laws themselves, so they all are convinced that 2^0 is 0.

How Many Seats? Lesson Study by Catherine Lewis. http://www.lessonresearch.net/howmanyseats.html
A wonderful lesson study cycle with lots of honest reflection by the teachers. The teachers shift to observing student thinking as opposed to teacher moves and grapple with the pitfalls of using tables to represent functions.

My Brown Eyes, by Jay Koh. http://www.master-comm.com/mbevideo.htm.
Film about a resourceful, independent Korean child having a horrible introduction to an American school that is not prepared for cultural difference. An entryway into discussing cultural assumptions.