Big Problem 7: Which Is Bigger? . Compare the cardinalities of the following sets:

1. $\mathbb{N}$
2. $\mathbb{R}$
3. $\mathbb{R}^2$
4. $\mathbb{R}^n$
5. set of functions from $\mathbb{N} \to \mathbb{N}$
6. set of functions from $\mathbb{R} \to \mathbb{R}$
7. algebraic numbers (i.e. the roots of finite degree polynomials with integer coefficients)
8. the Cantor Set
9. and the open interval $(0, 1)$
10. set of all things you can describe in finite English sentences

Big Problem 8: Tulie Numbers. Recall the Tulie Numbers class activity.

1. Show that every rational number is dull, i.e. not endless.
2. Show that every dull number is rational.
3. Explain why the last two statements imply that a number has an infinite continued fraction representation if and only if it is irrational.
4. Convince me that e is irrational by finding a pattern in its continued fraction expansion.

Big Problem 9: Irrational Numbers.

1. Use an argument similar to Euclid's method to prove $\sqrt[n]{2}$ is irrational for any natural number $n > 1$.

2. Use an argument similar to Euclid's method to prove that $\sqrt{p}$ is irrational for p prime.

3. Use the continued fractions method to prove the square root of n^2+1 is irrational for any natural number $n > 0$. (You can assume the results of Big Problem 8.)

Big Problem 10: Closed Form Formula for the Fibonacci Sequence.

1. We call a sequence A_n *Fibonacci-esque* if it has the property $A_{n+2} = A_{n+1} + A_n$.

2. Find all geometric sequences that are Fibonacci-esque. That is, all numbers C such that the sequence $A_n = kC_n$ is Fibonacci-esque (for constant k).

3. Prove linear combinations of Fibonacci-esque sequences have to be Fibonacci-esque.

4. Find a closed-form formula for the famous Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots$ by finding a sum of Fibonacci-esque geometric sequences that has the famous values.