1. **Decimal Fractions**.

   (a) Prove that every rational number has either a terminating or repeating expansion. (Hint: long division.)

   (b) In light of the last question, give me a clear way to generate an infinite decimal expansion that does not repeat, thus constructing an irrational number. Be sure I can tell how to generate each digit and also make an argument why it does not repeat.

   (c) Consider an infinite decimal expansion $0.d_1d_2d_3\ldots$. Write it as a limit of a sequence of truncated ("cut-off") decimal representations.

   (d) Prove that the sequence in the last question is Cauchy. (Reminder: a sequence a_n is *Cauchy* if for any $\epsilon > 0$, there is a natural number N such that if $m, n > N$, then $|a_m - a_n| < \epsilon$.)

   (e) (don't turn in) Remind yourself what a completion of a metric space is, and convince yourself that the real numbers are the completion of the rational numbers.

   (f) Write $0.999\ldots$ as the limit of a sequence of terminating decimals. Prove that the limit of this sequence is 1. Yes, you'll have to use an $\epsilon - N$ argument.

2. **Comparing Set Cardinalities**. Prove using the definition of "same size" and "fits in" in class that the following pairs of sets have the same size:

   (a) Prime numbers and whole numbers.

   (b) Terminating decimals and repeating decimals (here you can count an infinitely repeating 0 as repeating).