

1. (don't turn in) Symbolically check that American, European, Make It Nice and Negative Numbers methods work in base 4 using $311_4 - 112_4$ as an example.

2. (don't turn in) Demonstrate **why** Lattice Multiplication works using the example 78×23 .

3. **French Hand Multiplication.** In Butterworth's *What Counts*, p.205, he writes:

To this day [about 1930], the peasant of central France (Auvergne) uses a curious method for multiplying numbers above 5. If he wished to multiply 9×8 he bends down 4 fingers on his left hand (4 being the excess of 9 over 5) and 3 fingers on his right hand ($8 - 5 = 3$). Then the number of bent-down fingers gives him the tens of the result ($4 + 3 = 7$) while the product of the unbent fingers gives him the units ($1 \times 2 = 2$).

Does this method work for multiplying any two numbers between 6 and 9? Justify why, or give a counter-example.

4. **Doubling/Halving Multiplication.** I showed you in class how to multiply two numbers $A \times B$, by making two columns. In the first column, halve A (discarding the remainder), the next column, double B . Go to the next line and repeat the process. Stop when you halve A down to 1. Then you circle each line with an odd A . Add up all the numbers in the B column. Demonstrate why this works.
5. Take the natural numbers as the set we know and love equipped with addition and multiplication (and for simplicity, we count 0 as a natural number). For each natural number A let us define a new mysterious opposite number, $-A$, with the property $A + (-A) = 0 = (-A) + A$, i.e. $-A$ is an additive inverse of A . The union of the naturals and all their opposites we'll now call the integers. The nonzero natural numbers are called positive integers, and their opposites, negative integers.

- Prove from the definition that the opposite of $(-X)$ is X (or that "minuses cancel") and therefore every integer has an opposite.

6. We would like to extend addition and multiplication to these new opposites in a way that extends essential properties of natural number arithmetic. We ask that

- they obey the distributive law
- addition and multiplication are associative and commutative
- 0 is an additive identity
- 1 is a multiplicative identity

It can be checked that the integers can be assigned these properties in a way that doesn't lead to contradictions. You can now deduce a lot about the integers using **only these basic properties**.

- Prove that 0 is the unique additive identity (i.e. prove that any additive identity has to equal 0).
- Prove that every integer has a unique opposite.
- Prove zero times anything is zero.
- Prove the product of $-A$ and $-B$ is AB and therefore “negative times negative is positive”. (Hint: The slickest way I know proves first that $(-A)(-B)$ and $(A)(-B)$ are opposites, then that $(A)(-B)$ and AB are opposites, then wraps things up.)