1. (don't turn in) Symbolically check that American, European, Make It Nice and Negative Numbers methods work in base 4 using $311_4 - 112_4$ as an example.

2. (don't turn in) Demonstrate **why** Lattice Multiplication works using the example 78×23.

3. **French Hand Multiplication**. In Butterworth's *What Counts*, p.205, he writes:

   To this day [about 1930], the peasant of central France (Auvergne) uses a curious method for multiplying numbers above 5. If he wished to multiply 9×8 *he bends down 4 fingers on his left hand (4 being the excess of 9 over 5) and 3 fingers on his right hand* $(8 - 5 = 3)$ *. Then the number of bent-down fingers gives him the tens of the result* $(4 + 3 = 7)$ *while the product of the unbent fingers gives him the units* $(1 \times 2 = 2)$*.*

   Does this method work for multiplying any two numbers between 6 and 9? Justify why, or give a counter-example.

4. **Doubling/Halving Multiplication**. I showed you in class how to multiply two numbers $A \times B$, by making two columns. In the first column, halve A (discarding the remainder), the next column, double B. Go to the next line and repeat the process. Stop when you halve A down to 1. Then you circle each line with an odd A. Add up all the numbers in the B column. Demonstrate why this works.

5. Take the natural numbers as the set we know and love equipped with addition and multiplication (and for simplicity, we count 0 as a natural number). For each natural number A let us define a new mysterious opposite number, $-A$, with the property $A+(-A) = 0 = (-A)+A$, i.e. $-A$ is an additive inverse of A. The union of the naturals and all their opposites we'll now call the integers. The nonzero natural numbers are called positive integers, and their opposites, negative integers.

   - Prove from the definition that the opposite of $(-X)$ is X (or that "minuses cancel") and therefore every integer has an opposite.

6. We would like to extend addition and multiplication to these new opposites in a way that extends essential properties of natural number arithmetic. We ask that

- they obey the distributive law
- addition and multiplication are associative and commutative
- 0 is an additive identity
- 1 is a multiplicative identity

It can be checked that the integers can be assigned these properties in a way that doesn't lead to contradictions. You can now deduce a lot about the integers using **only these basic properties**.

(a) Prove that 0 is the unique additive identity (i.e. prove that any additive identity has to equal 0).

(b) Prove that every integer has a unique opposite.

(c) Prove zero times anything is zero.

(d) Prove the product of $-A$ and $-B$ is AB and therefore "negative times negative is positive". (Hint: The slickest way I know proves first that $(-A)(-B)$ and $(A)(-B)$ are opposites, then that $(A)(-B)$ and AB are opposites, then wraps things up.)