

Math 475 Homework 9

DUE APRIL 29TH, 2009

1. We've defined $e = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$. There is another famous series expression for e , which we can use to prove e is irrational.
 - (a) Write down the Taylor Series for the function e^x . (You can look it up. I'll assume you derived it in Calculus 2 and can prove it converges everywhere.)
 - (b) Write down an infinite series expression for e .
 - (c) Assuming e is genuinely equal to the infinite series, prove e is irrational. (Hint: if $e = p/q$, multiply through by $(q!)$. This should lead to an integer part plus something you can prove is less than 1.)
2. Prove $\log_2 5$ is irrational. (Hint: what if it were equal to p/q ?)