**Changes for Content of Mathematics Subject Matter Requirements (SMRs)**

**Between 2009 and 2013 Versions**

This document is a little toolkit for identifying changes between the old and new CTC SMRs. This could be useful for people who have to resubmit their old subject matter waiver program approvals.

(Pages 2—8) First, there is a Microsoft Word "track changes" version of the new tables. You can flip between Review/Final Showing Markup and Original Showing Markup if you want to see the changes. This version was created by Rebecca Parker at CTC and compiled by Carol Fry Bohlin.

(Pages 9—14) Second, there is a side-by-side table version of the changes (2009/2013 versions) which was created by Eric Hsu. CTC claims that small changes are very meaningful, so almost all wording changes are **highlighted and bold**.

Corrections welcome!

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**Document History**

* *Nov 5 2013. CFB adds more Parker documentation.*
* *Nov 4 2013. Original version.*

**1. Tracked** **Changes for Content of Mathematics Subject Matter Requirements (SMRs)**

**Between 2009 and 2013 Versions**

*The following is compiled from emails sent from Rebecca Parker (California Commission on Teacher Credentialing) to Carol Fry Bohlin on 28 October 2013:*

All approved Mathematics subject matter programs (programs that waive the CSET: Math) must review their courses of study to ensure the programs are teaching the new subject matter requirements (SMRs) essential to effectively teach students the new California Common Core State Standards (CCSS). Programs must be teaching the new SMRs this Fall (Fall 2013). In addition, programs must submit a new Alignment Matrix to the Commission on Teacher Credentialing (CTC) by June 2014. Please do not revise courses to reflect the students’ CCSS. The CTC convened expert panels to review the CCSS and identify the skills teachers must have in order to be able to effectively teach students the CCSS. (English SMPPs are making the same revisions.)

A revised alignment matrix is available on the (CTC) website at [www.ctc.ca.gov/educator-prep/STDS-subject-matter.html](http://www.ctc.ca.gov/educator-prep/STDS-subject-matter.html" \t "_blank). Once on the page, scroll down to find Mathematics. The link to the new Alignment Matrix is just below the word "Mathematics." The handbook is also available, but is being updated and, although it contains the updated alignment matrix, it doesn’t provide information to help complete the matrix.

To update the alignment matrix, for each domain and sub-domain on the matrix, identify the courses, assignments, assessments, etc. through which your students are learning the new SMRs. Since your program must be teaching the new SMRs this Fall (Fall 2013), you could reference those courses, assignments, etc. in the matrix. This task doesn’t require a major reorganization or a resubmission of your program. In fact, please, do not submit copies of syllabi or anything else as documentation. The completed alignment matrix will be your assurance to the CTC that your students are being prepared appropriately.

Faculty need to be very careful to notice the verbs and verb changes. The 2009 standards say “know” a lot. The 2013 standards say “demonstrate”. The revised SMRs put much greater emphasis on performance because the CCSS requires more mathematical practice; developing procedural fluency, etc. The following is taken from the revised Mathematics SMPP Handbook, Attachment to Standard 3 (about required content). The quote is from the K-12 California Common Core State Standards for Mathematics document at [www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf](http://www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf) :

The Common Core State Standards in Mathematics are comprised of two types of standards; eight practice standards that are identical for each grade level and content standards that differ at each grade level. Separating the two types of standards emphasizes both the importance of “knowing” the content and of “being able to do/use” mathematical knowledge. Overlapping the types of standards elucidates skills such as “habits of mind” and perseverance, which are critical to mathematics and to staying the course of ones education through difficult and challenging times. These are worthy goals for students beginning at the earliest levels.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: "adaptive reasoning, strategic competence, conceptual understanding…, procedural fluency…, and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy)."[1]

For guidance or more information, contact Rebecca Parker [rparker@ctc.ca.gov](mailto:rparker@ctc.ca.gov" \t "_blank). Submit the completed alignment matrix to [Accreditation@ctc.ca.gov](mailto:Accreditation@ctc.ca.gov" \t "_blank) .

| **Domains for Mathematics** | **Coursework, Assignments, Assessments** |
| --- | --- |
| Domain 1: Algebra |  |
| * 1. **Algebraic Structures**  1. Demonstrate knowledge of why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings) 2. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if *a* < *b* and *c* < 0, then *ac* > *bc*) 3. Demonstrate knowledge that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field 4. Identify and translate between equivalent forms of algebraic expressions and equations using a variety of techniques (e.g., factoring, applying properties of operations) 5. Justify the steps in manipulating algebraic expressions and solving algebraic equations and inequalities 6. Represent situations and solve problems using algebraic equations and inequalities |  |
| * 1. **Polynomial Equations and Inequalities**  1. Analyze and solve polynomial equations with real coefficients using:  * the Rational Root Theorem for polynomials with integer coefficients * the Conjugate Root Theorem for polynomial equations with real coefficients * the Binomial Theorem  1. Prove and use the Factor Theorem and the quadratic formula for real and complex quadratic polynomials 2. Solve polynomial inequalities |  |
| **1.3 Functions**   1. Analyze general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) and apply arithmetic operations on functions 2. Analyze properties of linear functions (e.g., slope, intercepts) using a variety of representations 3. Demonstrate knowledge of why graphs of linear inequalities are half planes and be able to apply this fact 4. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) 5. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) 6. Model and solve problems using nonlinear functions |  |
| **1.4 Linear Algebra**   1. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples 2. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) 3. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) 4. Analyze the properties of proportional relationships, lines, linear equations, and their graphs, and the connections between them 5. Model and solve problems using linear equations, pairs of simultaneous linear equations, and their graphs |  |

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| **Domain 2: Geometry** |  |
| **2.1 Plane Euclidean Geometry**   1. Apply the Parallel Postulate and its implications and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) 2. Demonstrate knowledge of complementary, supplementary, and vertical angles 3. Prove theorems, justify steps, and solve problems involving similarity and congruence 4. Apply and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, triangle inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) 5. Apply and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) 6. Identify and justify the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular polygons with 3, 4, 5, 6, and 8 sides) |  |
| **2.2 Coordinate Geometry**   1. Use techniques in coordinate geometry to prove geometric theorems 2. Model and solve mathematical and real-world problems by applying geometric concepts to two-dimensional figures 3. Translate between the geometric description and the equation for a conic section 4. Translate between rectangular and polar coordinates and apply polar coordinates and vectors in the plane |
| **2.3 Three-Dimensional Geometry**   1. Demonstrate knowledge of the relationships between lines and planes in three dimensions (e.g., parallel, perpendicular, skew, coplanar lines) 2. Apply and justify properties of three-dimensional objects (e.g., the volume and surface area formulas for prisms, pyramids, cones, cylinders, spheres) 3. Model and solve mathematical and real-world problems by applying geometric concepts to three-dimensional figures |  |
| **2.4 Transformational Geometry**   1. Demonstrate knowledge of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection), including their basic properties in relation to congruence 2. Demonstrate knowledge of dilations (e.g., similarity transformations or change in scale factor), including their basic properties in relation to similarity, volume, and area |  |

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| **Domain 3: Number and Quantity** |  |
| **3.1 The Real and Complex Number Systems**   1. Demonstrate knowledge of the properties of the real number system and of its subsets 2. Perform operations and recognize equivalent expressions using various representations of real numbers (e.g., fractions, decimals, exponents) 3. Solve real-world and mathematical problems using numerical and algebraic expressions and equations 4. Apply proportional relationships to model and solve real-world and mathematical problems 5. Reason quantitatively and use units to solve problems (i.e., dimensional analysis) 6. Perform operations on complex numbers and represent complex numbers and their operations on the complex plane | |
| **3.2 Number Theory**   1. Prove and use basic properties of natural numbers (e.g., properties of divisibility) 2. Use the principle of mathematical induction to prove results in number theory 3. Apply the Euclidean Algorithm 4. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple; show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime; prove that the square root of any number, not a perfect square number, is irrational) |  |

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| **Domain 4: Probability and Statistics** | | |
| **4.1 Probability**   1. Prove and apply basic principles of permutations and combinations 2. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles, sample space) 3. Use and explain the concepts of conditional probability and independence 4. Compute and interpret the probability of an outcome, including the probabilities of compound events in a uniform probability model 5. Use normal, binomial, and exponential distributions to solve and interpret probability problems 6. Calculate expected values and use them to solve problems and evaluate outcomes of decisions | |  |
| **4.2 Statistics**   1. Compute and interpret the mean and median of both discrete and continuous distributions 2. Compute and interpret quartiles, range, interquartile range, and standard deviation of both discrete and continuous distributions 3. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results 4. Apply the method of least squares to linear regression 5. Apply the chi-square test 6. Interpret scatter plots for bivariate data to investigate patterns of association between two quantities (e.g., correlation), including the use of linear models 7. Interpret data on a single count or measurement variable presented in a variety of formats (e.g., dot plots, histograms, box plots) 8. Demonstrate knowledge of P-values and hypothesis testing 9. Demonstrate knowledge of confidence intervals | |  |



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| **Domain 5: Calculus** |  |
| **5.1 Trigonometry**   1. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin2*x* + cos2*x* = 1 and that this identity leads to 1 + tan2*x* = sec2*x* and 1 + cot2*x* = csc2*x* 2. Prove and apply the sine, cosine, and tangent sum formulas for all real values 3. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems, using the unit circle) 4. Apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan) 5. Apply polar representations of complex numbers (e.g., DeMoivre's Theorem) 6. Model periodic phenomena with periodic functions 7. Recognize equivalent identities, including applications of the half-angle and double-angle formulas for sines and cosines |  |
| **5.2 Limits and Continuity**   1. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit 2. Show that a polynomial function is continuous at a point 3. Apply the intermediate value theorem, using the geometric implications of continuity |  |
| **5.3 Derivatives and Applications**   1. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative 2. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) 3. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule 4. Use the derivative to solve rectilinear motion, related rate, and optimization problems 5. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity) 6. Solve separable first-order differential equations and apply them to growth and decay problems |  |
| **5.4 Integrals and Applications**   1. Derive definite integrals of standard algebraic functions using the formal definition of integral 2. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums) 3. Prove the fundamental theorem of calculus, and use it to interpret definite integrals as antiderivatives 4. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures |  |
| **5.5 Sequences and Series**   1. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) 2. Determine convergence of a given sequence or series using standard techniques (e.g., ratio, comparison, integral tests) 3. Calculate Taylor series and Taylor polynomials of basic functions |  |

**2. Side-by-Side Changes for Content of Mathematics Subject Matter Requirements (SMRs) Between 2009 and 2013 Versions**

*Eric Hsu* [*erichsu@sfsu.edu*](mailto:erichsu@sfsu.edu)*, November 4th, 2013*

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| **NEW Common Core SMRs** | **OLD SMRs** |
| Domain 1:  Algebra | Domain 1. Algebra |
| 1.1 Algebraic Structures | 1.1 Algebraic Structures |
| a. **Demonstrate knowledge** of why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings) | a. **Know** why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings) |
| b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) | b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) |
| c. **Demonstrate knowledge** that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field | c. **Know** that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field |
| d. Identify and translate between equivalent forms of algebraic expressions and equations using a variety of techniques (e.g., factoring, applying properties of operations) |  |
| e. Justify the steps in manipulating algebraic expressions and solving algebraic equations and inequalities |  |
| f. Represent situations and solve problems using algebraic equations and inequalities |  |
| 1.2 Polynomial Equations and Inequalities | 1.2 Polynomial Equations and Inequalities |
|  |  |
| a. **Analyze and solve** polynomial equations with real coefficients using: | b. **Prove and use** the following: |
| • the Rational Root Theorem for polynomials with integer coefficients | • the Rational Root Theorem for polynomials with integer coefficients |
| • the Fundamental Theorem of Algebra | (1.2.c). Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra |
| • the Conjugate Root Theorem for polynomial equations with real coefficients | • the Conjugate Roots Theorem for polynomial equations with real coefficients |
| • the Binomial Theorem | • the Binomial Theorem |
| b. Prove and use the Factor Theorem and the quadratic formula for real and complex quadratic polynomials | • the Factor Theorem, • the Quadratic Formula for real and complex quadratic polynomials |
| c. Solve polynomial inequalities |  |
| 1.3 Functions | 1.3 Functions |
| a. Analyze general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) and apply arithmetic operations on functions | a. Analyze **and prove** general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) |
| b. Analyze properties of linear functions (e.g., slope, intercepts) using a variety of representations |  |
| c. **Demonstrate knowledge** of why graphs of linear inequalities are half planes and be able to apply this fact | (1.2.a) **Know** why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming) |
| d. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) |
| e. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) |
| f. Model and solve problems using nonlinear functions |  |
| 1.4 Linear Algebra | 1.4 Linear Algebra |
| a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples | a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) **and cross products** |
| b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) | b. Prove the basic properties of vectors  (e.g., perpendicular vectors have zero dot product) |
| c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) | c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) |
| d. Analyze the properties of proportional relationships, lines, linear equations, and their graphs, and the connections between them |  |
| e. Model and solve problems using linear equations, pairs of simultaneous linear equations, and their graphs |  |
| Domain 2:  Geometry | Domain 2. Geometry |
| **2.1 Plane Euclidean Geometry** | **2.1 Parallelism** |
| a. Apply the Parallel Postulate and its implications and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) | a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) |
| b. **Demonstrate knowledge** of complementary, supplementary, and vertical angles | b. **Know** that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic) |
|  | 2.2 Plane Euclidean Geometry |
| c. Prove theorems, **justify steps,** and solve problems involving similarity and congruence | a. Prove theorems and solve problems involving similarity and congruence |
| d. Apply and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, triangle inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | b. **Understand,** apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) |
| e. Apply and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) | c. **Understand,** apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) |
| f. **Identify and justify** the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular polygons with 3, 4, 5, 6, and 8 sides) | d. **Justify and perform** the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8) |
| 2.2 Coordinate Geometry |  |
| a. Use techniques in coordinate geometry to prove geometric theorems | e. Use techniques in coordinate geometry to prove geometric theorems |
| b. Model and solve mathematical and real-world problems by applying geometric concepts to two-dimensional figures |  |
| c. Translate between the geometric description and the equation for a conic section |  |
| d. Translate between rectangular and polar coordinates and apply polar coordinates and vectors in the plane |  |
| 2.3 Three-Dimensional Geometry | 2.3 Three-Dimensional Geometry |
| a. Demonstrate knowledge of **the relationships between lines and planes in three dimensions** (e.g., parallel, perpendicular, skew, coplanar lines) | a. Demonstrate an understanding of **parallelism and perpendicularity of lines and planes in three dimensions** |
| b. Apply and justify properties of three-dimensional objects (e.g., the volume and surface area formulas for prisms, pyramids, cones, cylinders, spheres) | b. **Understand,** apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., **derive the** volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres) |
| c. Model and solve mathematical and real-world problems by applying geometric concepts to three-dimensional figures |  |
| 2.4 Transformational Geometry | 2.4 Transformational Geometry |
| a. Demonstrate knowledge of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection), including their basic properties in relation to congruence | a. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection) |
| b. **Demonstrate knowledge** of dilations (e.g., similarity transformations or change in scale factor), including their basic properties in relation to similarity, **volume, and area** | b. **Understand and prove** the basic properties of dilations (e.g., similarity transformations or change of scale) |
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| **Domain 3:  Number and Quantity** | **Domain 3. Number Theory** |
| 3.1 The Real and Complex Number Systems |  |
| a. Demonstrate knowledge of the properties of the real number system and of its subsets |  |
| b. Perform operations and recognize equivalent expressions using various representations of real numbers (e.g., fractions, decimals, exponents) |  |
| c. Solve real-world and mathematical problems using numerical and algebraic expressions and equations |  |
| d. Apply proportional relationships to model and solve real-world and mathematical problems |  |
| e. Reason quantitatively and use units to solve problems (i.e., dimensional analysis) |  |
| f. Perform operations on complex numbers and represent complex numbers and their operations on the complex plane |  |
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| 3.2 Number Theory | 3.1 Natural Numbers |
| a. Prove and use basic properties of natural numbers (e.g., properties of divisibility) | a. Prove and use basic properties of natural numbers (e.g., properties of divisibility) |
| b. Use the principle of mathematical induction to prove results in number theory | b. Use the Principle of Mathematical Induction to prove results in number theory |
| c. Apply the Euclidean Algorithm | c. **Know and** apply the Euclidean Algorithm |
| d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple; show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime; prove that the square root of any number, not a perfect square number, is irrational) | d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational) |
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| Domain 4:  Probability and Statistics | Domain 4. Probability and Statistics |
| 4.1 Probability | 4.1 Probability |
| a. Prove and apply basic principles of permutations and combinations | a. Prove and apply basic principles of permutations and combinations |
| b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles, sample space) | b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles) |
| c. Use and explain the concepts of conditional probability **and independence** | c. Use and explain the concept of conditional probability |
| d. Compute and interpret the probability of an outcome, **including the probabilities of compound events in a uniform probability model** | d. Interpret the probability of an outcome |
| e. Use normal, binomial, and exponential distributions to solve and interpret probability problems | e. Use normal, binomial, and exponential distributions to solve and interpret probability problems |
| f. Calculate expected values and use them to solve problems and evaluate outcomes of decisions |  |
| 4.2 Statistics | 4.2 Statistics |
| a. Compute and interpret the mean and median of both discrete and continuous distributions | a. Compute and interpret the mean, median, **and mode** of both discrete and continuous distributions |
| b. Compute and interpret quartiles, range, **interquartile range,** and standard deviation of both discrete and continuous distributions | b. Compute and interpret quartiles, range, **variance,** and standard deviation of both discrete and continuous distributions |
| c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results | c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results |
| d. **Apply** the method of least squares to linear regression | d. **Know** the method of least squares and apply it to linear regression **and correlation** |
| e. Apply the chi-square test | e. **Know and** apply the chi-square test |
| f. Interpret scatter plots for bivariate data to investigate patterns of association between two quantities (e.g., correlation), including the use of linear models |  |
| g. Interpret data on a single count or measurement variable presented in a variety of formats (e.g., dot plots, histograms, box plots) |  |
| h. Demonstrate knowledge of P-values and hypothesis testing |  |
| i. Demonstrate knowledge of confidence intervals |  |
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| Domain 5: Calculus | Domain 5. Calculus\* |
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| 5.1 Trigonometry | 5.1 Trigonometry |
| a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin^2 x + cos^2 x = 1 and that this identity leads to 1 + tan^2 x = sec^2 x and 1 + cot^2 x = csc^2 x | a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin^2 x + cos^2 x = 1 and that this identity leads to 1 + tan^2 x = sec^2 x and 1 + cot^2 x = csc^2 x |
| b. Prove and apply the sine, cosine, and tangent sum formulas for all real values | b. Prove the sine, cosine, and tangent sum formulas for all real values… |
| c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems, using the unit circle) | c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) |
| d. Apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan) | d. **Know and** apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan) |
| e. Apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | e. Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) |
| f. Model periodic phenomena with periodic functions |  |
| g. **Recognize equivalent identities,** including applications of the half-angle and double-angle formulas for sines and cosines | (5.1.b). ...and derive special applications of the sum formulas (e.g., double angle, half angle) |
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| 5.2 Limits and Continuity | 5.2 Limits and Continuity |
| a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit | a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit |
| b. Show that a polynomial function is continuous at a point | b. Show that a polynomial function is continuous at a point |
| c. Apply the intermediate value theorem, using the geometric implications of continuity | c. **Know and** apply the Intermediate Value Theorem, using the geometric implications of continuity |
| 5.3 Derivatives and Applications | 5.3 Derivatives and Applications |
| a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative | a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative |
| b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) | b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton’s method, and instantaneous rate of change) |
| c. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule | c. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle’s Theorem, the Mean Value Theorem, and L’Hopital’s rule |
| d. Use the derivative to solve rectilinear motion, related rate, and optimization problems | d. Use the derivative to solve rectilinear motion, related rate, and optimization problems |
| e. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity) | e. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity) |
| f. Solve separable first-order differential equations and apply them to growth and decay problems | f. Solve separable first-order differential equations and apply them to growth and decay problems |
|  |  |
| 5.4 Integrals and Applications | 5.4 Integrals and Applications |
| a. Derive definite integrals of standard algebraic functions using the formal definition of integral | a. Derive definite integrals of standard algebraic functions using the formal definition of integral |
| b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums) | b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums) |
| c. Prove the fundamental theorem of calculus, and use it to interpret definite integrals as antiderivatives | c. Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives |
| d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures |
|  |  |
| 5.5 Sequences and Series | 5.5 Sequences and Series |
| a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) | a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) |
| b. Determine convergence of a given sequence or series using standard techniques (e.g., ratio, comparison, integral tests) | b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests) |
| c. Calculate Taylor series and Taylor polynomials of basic functions | c. Calculate Taylor series and Taylor polynomials of basic functions |
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|  | Domain 6. History of Mathematics\* |
|  | 6.1 Chronological and Topical Development of Mathematics |
|  | a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society |
|  | b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender |