

We can now try this same procedure to create our own Escher-like tessellating shape. There are two ways to approach our modification of a polygon to create a non-polygonal tessellating shape. One way is to have some specific object in mind—such as a horse—and to modify the sides of the polygon until its contour resembles that object. A second approach is to modify the sides of the polygon with random curves, then interpret the resulting shape by adding details to highlight its interior.

In figure 7-7, we see a square that has been modified by translation (with no specific object in mind). Figure 7-8 shows the modified square interpreted first as a witch and then, with the shape rotated counterclockwise 90°, as a winged seahorse.

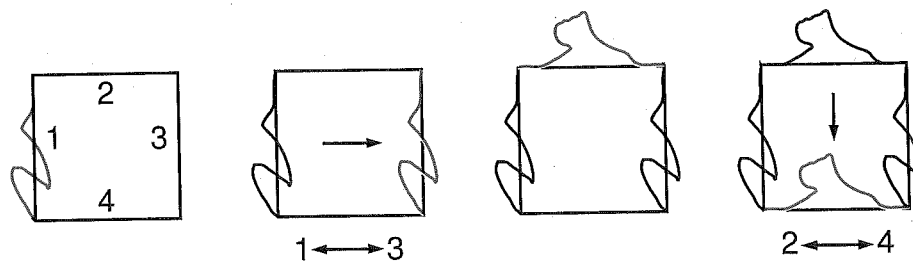
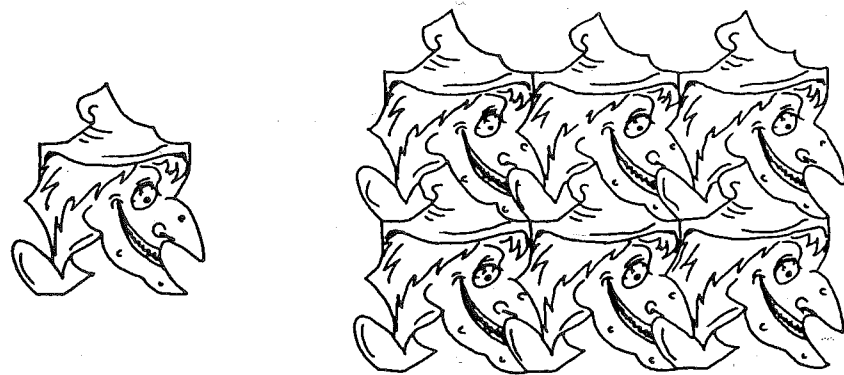
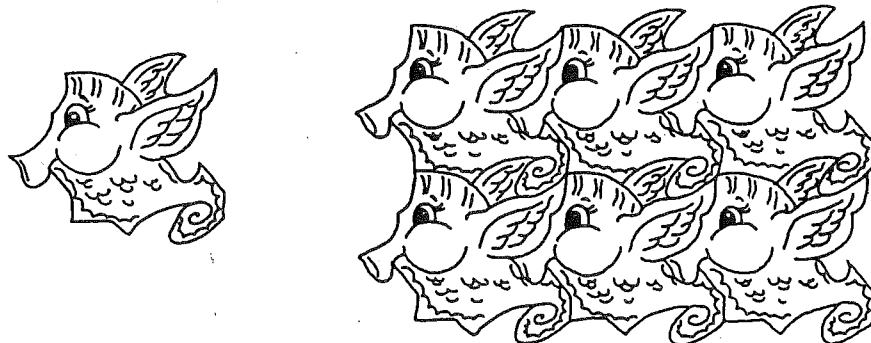


Fig. 7-7. Modifying by translation



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Fig. 7-8. Alternative interpretations of the same tessellating shape

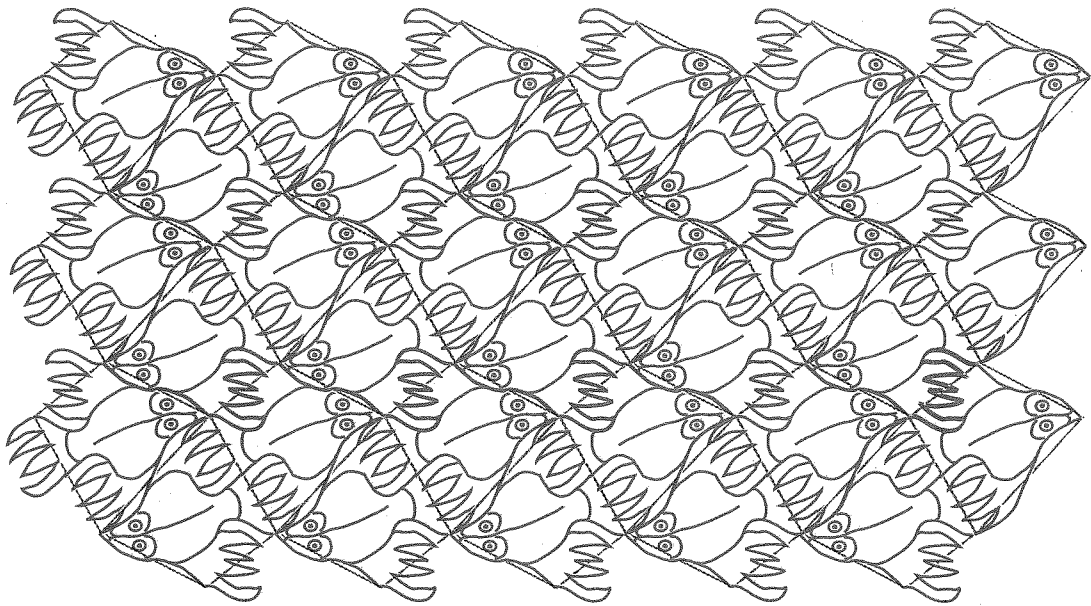


Fig. 7-14. A grid of scalene quadrilaterals underlying the *Fish* design

Note that although the orientation of the quadrilaterals alternates, each fish fits in its quadrilateral in exactly the same way as all the others. Let's investigate how we might create this fish shape from the original quadrilateral (see figure 7-15). We number the sides of that quadrilateral and study the contour of the fish inside it. Looking at each side separately, we see a type of change familiar from chapter 5: modifying by rotation about the midpoint of a side. That is, we modify a half-side of side 1 and rotate our modification 180° about the midpoint of that side. As a result, a hole on half of side 1 becomes a congruent bump on the other half. As you can see in figure 7-15, the same is true of all four sides.

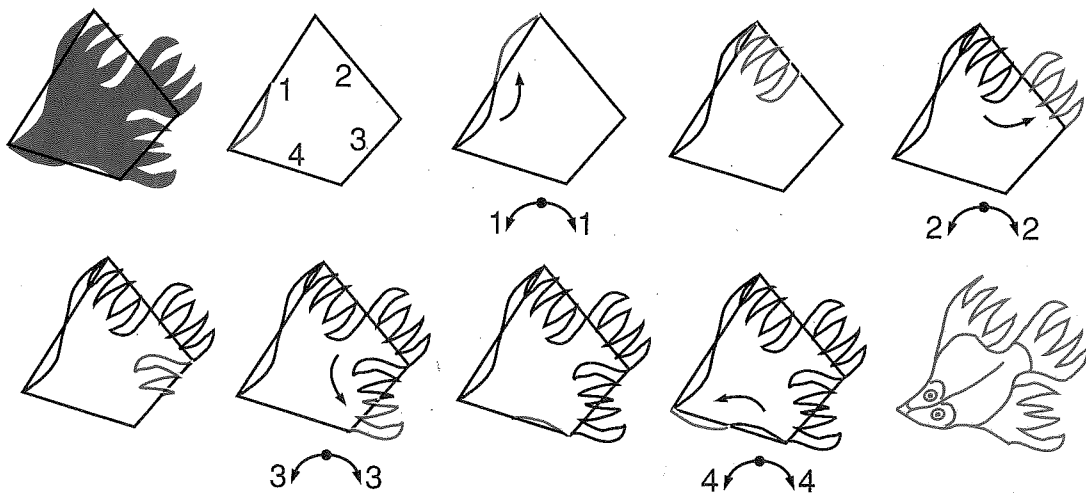


Fig. 7-15. Modifying by rotation about midpoints of sides

Any quadrilateral will tessellate if we rotate it 180° about the midpoint of each of its sides. The four angles of the quadrilateral, totalling 360° , then surround each vertex in the tessellation, as shown in figure 7-16.

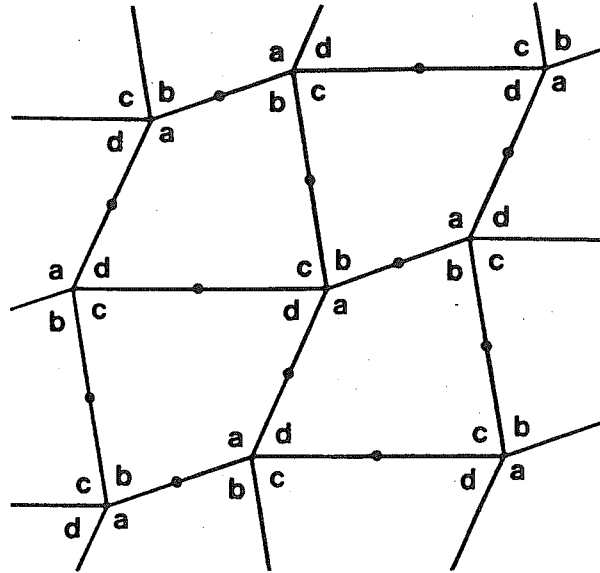


Fig. 7-16. The orientation of a quadrilateral in its tessellation

Given the enormous variety of quadrilaterals available to us, we can use rotation about midpoints of sides to create a wealth of Escher-like tessellating shapes. Each resulting tessellation will have four different centers of two-fold rotation: the midpoints of the four sides of each tessellating shape. Figures 7-17 through 7-21 demonstrate some of the possibilities.

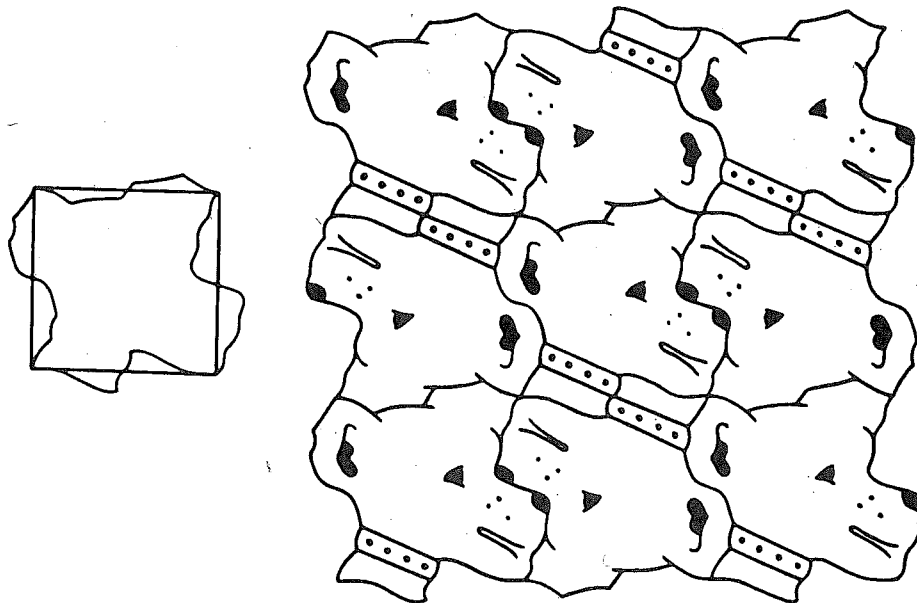
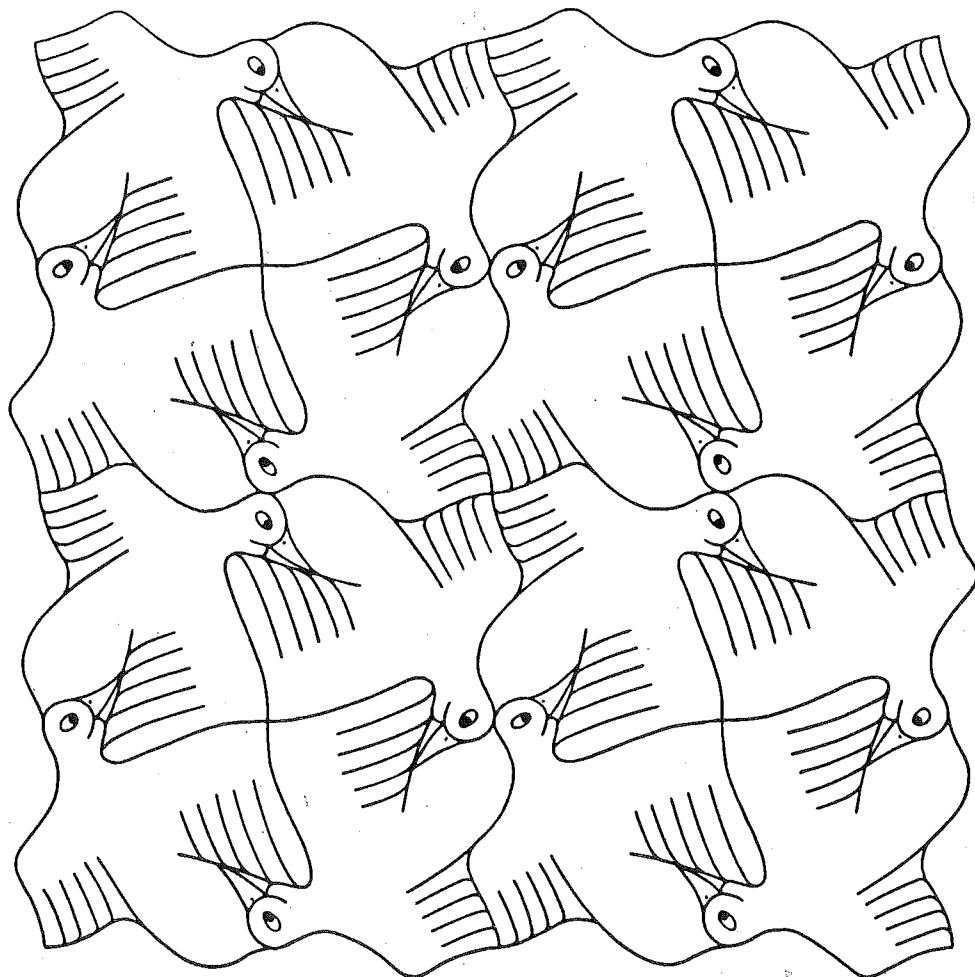
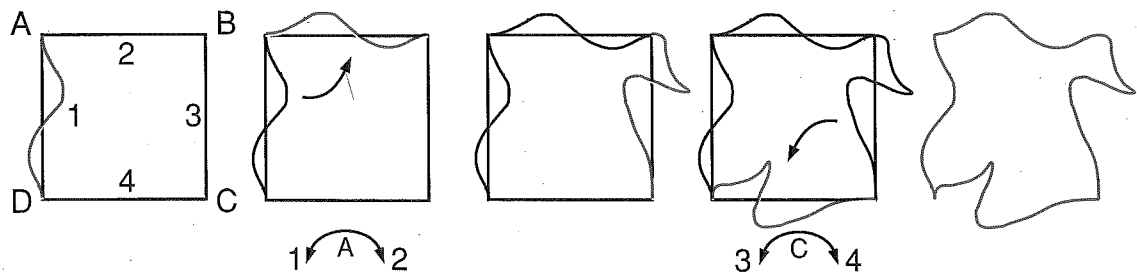


Fig. 7-17. A square modified by rotation about midpoints of sides

Now let's try to create our own tessellating shape by modifying two opposite sides of a square and rotating each modification about a vertex to an adjacent side. This is what we have done in figure 7-33. We started with a bird in mind and modified the sides of the square until its contour resembled a bird with wings outspread. The resulting tessellation has two centers of four-fold rotation and two centers of two-fold rotation, as does the Escher tessellation whose pattern inspired it.

Fig. 7-33. Modifying by rotation about vertices



Designing an Escher-like shape and drawing its tessellation is a time-consuming yet satisfying exercise. Following are some practical tips to help you get started.

A scalene quadrilateral allows you the greatest freedom and flexibility when you attempt your first design. As you make your preliminary sketches, you may find that you want to alter the shape of your original polygon—and with a scalene quadrilateral, you can. However long its sides and whatever the size of its angles, a quadrilateral will tessellate the plane if each of the sides is rotated 180° about its midpoint. (The only drawback of a scalene quadrilateral is the relative difficulty of drawing its tessellating grid, but if you tessellate by the first method we suggest below, this is not a problem.)

Once your preliminary sketch is completed, you must prepare an accurate version of the shape for your tessellation. One simple and practical approach is to cut the original polygon shape from construction paper or lightweight cardboard, then cut appropriate “holes” and tape them on as corresponding “bumps” to represent your modifications. (In the case of glide reflection, you will need to flip the bump before taping.) You can mark the location of simple details with slits or small holes in your final shape.

To create the tessellation, position your pattern on another sheet of paper, trace about its perimeter, and mark the location of interior details. By repositioning and tracing the pattern again and again, you will see the tessellation evolve before your eyes.

A more precise procedure involves no cutting and allows you more freedom in adding interior details—but it also requires more time and patience, and either a light table or a window that you can draw against. You will need three sheets of translucent paper. Draw your polygon on two of these. Then, on one polygon, draw your modifications to the sides in *one* of their two locations. Tape this polygon to a window or light table and superimpose the second polygon precisely on top. Trace your initial modifications, then move the superimposed sheet as needed to locate and trace each modification in its new location. (For glide reflection, you will need to flip the base polygon.) You end up with an accurately drawn shape to serve as your pattern, to which you can add interior details at will.

Now you need to create the underlying grid for your tessellating polygon. To avoid the problem of grid lines in your finished design, we suggest drawing the *mirror image* of the grid on the *back* of the third sheet of translucent paper. That way, when you tape this sheet

face up over your pattern at the window or a light table, you can see the grid lines to help you position your shape, but they will not appear in your final drawing. (Alternatively, of course, you could draw the grid on a separate sheet and draw your final design on yet another sheet placed over it—but three sheets of paper start to get opaque, even with a light table.)

From here on, completing the drawing is simply a matter of meticulous tracing. You align a polygon in the grid precisely with the polygon of your pattern, then trace all marks *except the polygon* on the top sheet. Select an adjacent polygon, turn the sheet to align with the pattern, and trace—continuing in this manner until you are satisfied with the extent of your tessellation.

For either method—tracing around a cardboard pattern or through translucent paper—ink gives you better contrast than pencil, but slips and errors are harder to repair. If you use pencil, you can improve the contrast as follows: Photocopy your final drawing, process it at a slow setting through a thermal transparency maker, then photocopy the resultant acetate. The improvement in quality can be quite dramatic.

Whether you undertake the creation of an Escher-like drawing or turn to Escher's own art to further analyze his tessellations, the fundamental procedures that we have presented in this chapter give you all the necessary tools for these exciting investigations. We hope we have inspired you to explore further.