Each day, Dawn Selnes, a fifth-grade teacher in New York City, does a short minilesson on computation strategies. She usually chooses a string of five or six related problems (like the ones provided in this resource unit) and asks the students to solve them, one at a time, sharing their strategies with each other. She allows her students to construct their own strategies by decomposing numbers in ways that make sense to them. Posted around the room are signs the students made throughout the year as they developed a repertoire of strategies for operations with fractions. The signs read, "Make use of money," "Use clocks," "Halve and double," "Eliminate the fractions."

On the chalkboard today is the first problem in Dawn's string:

9 × 30

15 × 18

4 ½ × 60

2 ½ × 120

15 × 36

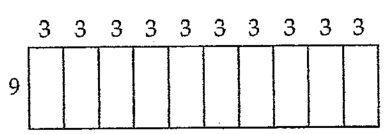
15 ½ × 36

5 ¼ × 40

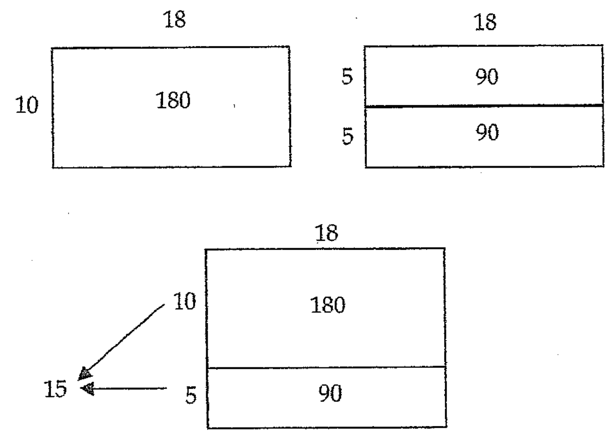
0.8 × 80

Although the string progresses to fractions and decimals, it begins with a few whole number multiplication problems, and one student, Alice, describes how she solved 9 × 30. "I just used all the factors," she explains. "I thought of it as 9 times 3 times 10. I knew that 9 times 3 is 27, and 27 times 10 is 270."

Dawn draws ten 9 × 3 arrays (showing how they fit together to make a 9 × 30) to represent her strategy and then asks for other strategies.

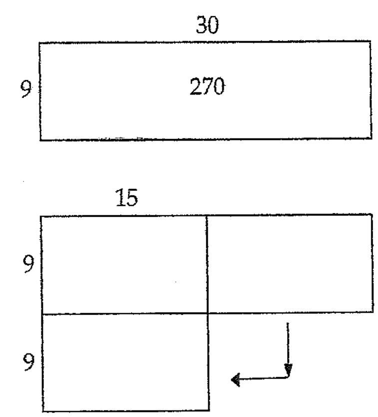


Most of the students have treated the problem similarly, so Dawn goes to the next problem in the string, 15 × 18. Several students use the distributive property here. Tom's strategy is representative of many, and several students nod in agreement as he explains how he multiplied 10 × 18 and got 180 and then took half of that to figure out the answer to 5 × 18. He completes the calculation by adding 180 to 90, for an answer of 270. Dawn draws these arrays:



Lara's strategy is similar, perhaps not as elegant, but it makes sense to her. She multiplies using tens, too, but she breaks up the 18 instead of the 15 and multiplies 10 × 15, and then 8 × 15. These two products together also result in 270, and Dawn represents them with an array model.

Ned agrees with that answer but with a smile he says, "Yeah, but you didn't even have to calculate. It's the same as 9 × 30, because the 30 is halved, and the 9 is doubled!" Dawn draws an array, cuts it in half, and the students discuss how the pieces can be moved to show the equivalence.



Although all the students in the class are comfortable with this doubling and halving strategy and understand why it works, they have not all thought to use it, because Dawn has turned the numbers around. It might have been more obvious if she had written 18 × 15 directly underneath 9 × 30. But she wants to challenge them to think.

Now Dawn moves to fractions. She writes 4 ½ × 60 as the third problem. Several students immediately raise their hands, but Dawn waits for those still working to finish. Alice is one of them, so she asks her to share first. "I split it into 4×60 first," Alice begins, "and I did that by doing $4 \times 6 = 24$. Then 24 times 10 is 240. Then I knew that a half of 60 is 30. So 30 plus 240 is 270."

"My way is kind of like yours," another classmate, Daniel, responds, "but I subtracted."

"But then you would get the wrong answer," Alice tells him, looking puzzled.

"No, what I mean is I did $5 \times 6 \times 10$. That was 300. Then I subtracted the 30."

"Where did you get the 5?" Several of his classmates are also now puzzled.

"That was easier for me than $4\frac{1}{2}$. But that's why I took 30 away at the end," Daniel explains, very proud of his strategy.

Dawn draws the array and checks to see whether everyone understands by asking who can explain Daniel's strategy in their own words. Several students do so, and Dawn seems satisfied that the group understands. "That's a really neat strategy, isn't it?" Daniel beams, and Dawn turns to Ned, "And what did you do, Ned? Your hand was up so quickly. Did you see a relationship to another problem again?"

Ned laughs and says, "Yep. Just doubling and halving again. It's the same as 9×30. The 9 was halved and the 30 was doubled."

Several students exclaim in surprise. Dawn smiles and goes to the next problem: $2\frac{1}{2} \times 120$. This time everyone's hand is up quickly, and Dawn calls on Tanya, who has not yet spoken. Tanya, like the rest of the class, has made use of the doubling and halving relationships in this string of problems.

The other strategies that have previously been discussed are also powerful strategies, and Dawn does not want to imply that they should be replaced by doubling and halving. She is only trying to help her students think about relationships in problems, to look to the numbers first before calculating. To ensure that this happens, she follows with the next two problems: 15×36, then $15\frac{1}{2} \times 36$. Most students see the relationship between the first one and 30×18. Since they have already calculated 15×18, they know they just need to double that answer. A few students solve it by doing 10×36 to get 360, halving that to get 180, and then adding these partial products for an answer of 540. For $15\frac{1}{2} \times 36$ everyone uses the distributive property, adding 18 more for an answer of 558.

Dawn moves next to a more difficult problem, challenging her students to generalize: $5\frac{1}{4} \times 40$. She asks the students to write their strategy and solution in their math journal, and then to turn to the person sitting next to them and share it. Most of the students use the distributive property, but a few double and halve to make $10\frac{1}{2} \times 20$, and then double and halve again: $21 \times 10 = 210$. "Do you think there is any way we could have done something like that but with one step only? Could we eliminate the fraction in one step?" Dawn asks them to ponder with pair talk. After a few minutes, she starts the discussion: "Lynn?"

"Maybe we could times the $5\frac{1}{4}$ by 4, and then take a quarter of the 40?" Lynn offers somewhat tentatively.

Dawn writes, $5\frac{1}{4} \times 4 \times 10$. "Hmm...let's look at what Lynn is saying." Adding parentheses, she writes $(5\frac{1}{4} \times 4) \times 10 = 5\frac{1}{4} \times (4 \times 10)$.

Rich discussion ensues until a consensus is reached that Lynn's strategy works, and then Dawn asks the class to consider whether it might work for decimals, too. She writes the last problem in the string: 0.8×80. With elegant efficiency, many of the students write: $0.8 \times 10 \times 8 = 8 \times 8$. At the end of the minilesson, a new sign is made for the wall of strategies. Dawn asks the students to describe the strategy in their own words and they decide to write, "Group the factors in pretty ways."

These young mathematicians are composing and decomposing numbers flexibly as they multiply rational numbers. They are inventing their own strategies. They are looking for relationships among the problems. They are looking carefully at the numbers first before they decide on a strategy.

Students don't do this automatically. Dawn has developed this ability in her students by focusing on computation during minilessons, with strings of related problems every day. She has developed the big ideas and models through investigations, but once this understanding has been constructed, she promotes fluency with computation strategies in minilessons such as this one.

Using Models during Minilessons

As you work with the minilessons from this unit, you will want to use the related models to depict students' strategies. The money, clock, and double open