Applying Diagramming to Division of Fractions Problems

“Division by fractions, the most complicated operation with the most complex numbers, can be considered as a topic at the summit of arithmetic.”


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Which Diagram Represents 6 ÷ 2?

Assume that all the small rectangles are congruent

Let’s call these *strip diagrams*. Some other types are percent bars and number lines.
Two Models for $6 \div 2$

Top diagram is the Quotative or Measurement Model:
How many 2’s are in 6? Answer: 3
The Measurement Model Extends Easily to Fractions

6 ÷ 2: How many 2’s are there in 6? Answer: 3

6 ÷ (1/2): How many ½’s are there in 6?
The Measurement Model Extends Easily to Fractions

6 ÷ 2: How many 2’s are there in 6?  Answer: 3

6 ÷ (1/2): How many ½’s are there in 6?  
Answer: 12
Language Issue

6 divided in half is ?

6 divided by a half is ?
The Partitive Model Does **Not** Extend Easily to Fractions

Bottom diagram is the Partitive Model:

6 ÷ 2: If 6 is divided into 2 equal parts, how much is in each part? Answer: 3

6 ÷ (1/2): If 6 is divided into ½ equal parts, how much is in each part???
Trick Used by Mathematicians: Reframe the Partitive Model So That It Does Extend to Fractions

Original: $6 \div 2$: If 6 is divided into 2 equal parts, how much is in each part? Answer: 3

Reframing: 2 of what make 6? Answer: Two 3’s
Apply Reframed Partitive Model to \( 6 \div \left( \frac{1}{2} \right) \)

\[ 6 \div 2: \text{2 of what make 6?} \]
Answer: \( 3 \)

\[ 6 \div \left( \frac{1}{2} \right): \text{½ of what makes 6?} \]
Answer: \( 12 \)
Apply Reframed Partitive Model to $6 \div (1/2)$

$6 \div 2$: 2 of what make 6? Answer: 3

$6 \div (1/2)$: $\frac{1}{2}$ of what makes 6? Answer: 12

This works, but...

BOTTOM LINE: Most people find the measurement model to be an easier way to think about division of fractions.
Draw a Diagram to Illustrate this Problem Situation

Work individually!

3/7 of the apples in a box are red apples. The rest are green apples. There are 32 green apples.
With a partner, think of a question. Then use/add to your diagram to help answer it.

3/7 of the apples in a box are red apples. The rest are green apples. There are 32 green apples.
Draw a Diagram to Illustrate this Division

Work individually first!

\[
\begin{array}{c}
\frac{3}{4} \div \frac{1}{2}
\end{array}
\]
What mistake might students make? How would you respond to this mistake?

\[
1 \frac{3}{4} \div \frac{1}{2}
\]
Make up a word problem whose solution involves this computation

\[
\frac{3}{4} \div \frac{1}{2}
\]
Use a Diagram to Solve this Problem

The Pillowcase Problem

In Ms. Smith’s sewing class, students are making pillowcases for the open house exhibit. Ms. Smith bought 10 yards of fabric for her class project. Each pillowcase requires $\frac{3}{4}$ yard of fabric. How many pillowcases can be cut from the fabric?

From an article in Mathematics Teaching in the Middle School, by Marcela D. Perlwitz. Pages 278-283. Copyright © 2005 The National Council of Teachers of Mathematics, Inc.
Make up a Problem that Could be Solved Using this Diagram
Make up a Problem whose Solution Involves Division of Fractions.

Then Use a Diagram to Solve it.