that involves active learning.

Conclusion

Teaching for understanding must begin with three basic steps: eliciting from the child what his/her understanding about a concept is, remediating any misconceptions, and using active learning to build conceptual understanding. In order for a child to successfully construct mathematics knowledge, the teacher has to help the child examine his/her existing schema, then help the child verbalize his/her understanding of the concepts, and finally acknowledge and thoroughly explore any misconceptions. Direct instruction may not expose these underlying weaknesses, and knowledge that is constructed as a result of teacher-led instruction could be based upon incorrect understandings. When children search for meaning in mathematics and/or try to keep the rules for different processes straight in their heads they will revert to incorrect algorithms that could yield the sucker bet.

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Understanding Fractions through Measurement

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The difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics. . ."
—Report of the National Math Panel, March 2008

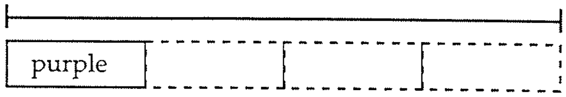
We all know that fractions are hard to teach and are difficult for our students to learn. As we know, understanding fractions is important for success in the secondary grades and beyond. Many people have called for students to spend more time developing a sense of fractions before moving to operations involving fractions. The goal of this article is to share a measurement approach for helping students develop their fraction sense.

Why a Measurement Approach

Fractions occur naturally in measurement activities since, when measuring, the item being measured is often not a complete iteration of the measurement unit. For example, a door may be two and a half feet wide; a melon may weigh two and a quarter pounds; your gas tank may need 15.34 gallons of gas. We encounter fractions everyday in these and other contexts. During measurement activities in the classroom students are often encouraged to round to the nearest whole unit; however, more precise measurements require the use of fractional values.

Advantages of a Measurement Approach

✔ *Presents fractions as a unit of measure.* In order to help students develop fraction sense, a measurement approach can be used to give meaning to fraction notation. In the following figure, the purple rod is 1/4 of the line segment because it takes 4 iterations of the purple rod to measure the line segment. By definition, the length of the purple rod is 1/4 of the length of the segment even if there is only one purple rod.

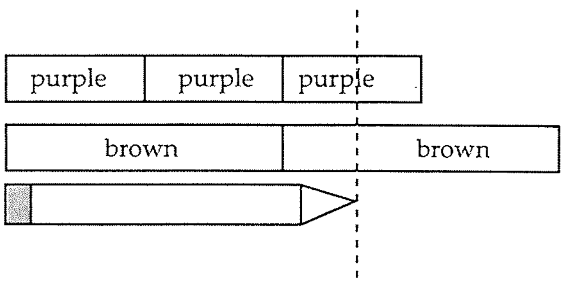


✔ *Highlights the importance of the unit.* In the figure above, the meaning of the purple rod as "1/4" exists only in relation to the line segment (the unit). This helps students understand that 1/4 implies a relationship, not just a name for the rod.

✔ *Supports connections between a fraction and a point on a number line.* A measurement approach supports the idea that all fractions can be represented as points on the number line, which is critical to the development of fraction sense. We want students to be able to understand fractions as numbers, and operate on and with fractions, without needing to consider pizzas, brownies, or marbles.

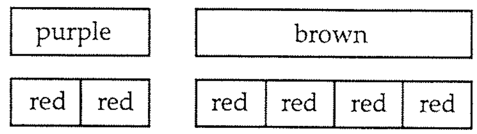
Measuring with Cuisenaire Rods

Cuisenaire Rods are a useful tool for engaging students with measurement activities. These rods are wooden (or plastic) three-dimensional blocks, and come in color-coded, graduated lengths from 1 cm to 10 cm. By using different length rods as the measuring unit, students find that the value of fractional lengths is based upon part-whole relations and may change depending upon which rod is considered the unit.



In looking at the pencil measurement task shown above, the challenge for students is to figure out how long the pencil is in purple rods and again in brown rods. The pencil is clearly more than 2 purple rods long and less than 3 purple rods long. What do we call this leftover part or remainder length? Is the remainder length 1/4 of a purple rod or 1/3 or 1/2? What about in relation to the brown rod? Is the remainder length 1/4 of a brown rod or 1/3 or 1/2? Can the same remainder length be 1/2 of one rod and 1/4 of another?

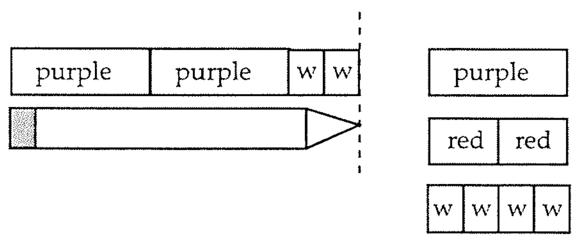
After students find a smaller rod that is the same length as the remainder length, the smaller rods can be iterated along the rod used as the measure unit, in this case the purple rod and brown rod, to determine the value of the remainder length.



In this way, students find that the remainder length, as measured by the red rod, is 1/2 of the purple rod and 1/4 of the brown rod. Thus the pencil is 2 1/2 purple rods long and 1 1/4 brown rods long.

Supporting understanding of equivalence

In addition to understanding the importance of the unit, a measurement approach can also support students' understanding of fraction equivalence. In the example described above, students can find that the remainder length can also be measured by two smaller (white) rods. As shown below, by iterating the white rods along the purple rod, it can now be determined that the white rods are fourths of the purple rod and the pencil is 2 2/4 purple rods long. This helps students to see that the pencil is both 2 1/2 purple rods long and 2 2/4 purple rods long, thus supporting their understanding of the equivalency between 1/2 and 2/4.



Tasks to try with your students

Choose items around your classroom that lend themselves to measurement with Cuisenaire Rods. Ask students: How long is the item in brown rods? How long is the item in purple rods? Students can use a table such as *Table 1* at the top of the next page to record their findings. The goal is to be as precise as possible, but we acknowledge that measurement is always estimation.

Item	Length in purple rods	Length in brown rods
Your pencil		
Mathematics book		
Marker		
Literature book		
Desk width		
Orange rod		

Editor's Note

If your students have access to computers, they can interact with a computer simulation to model fractions using Cuisenaire Rods at **learner.org/channel/courses/learningmath/number/session8/part_b/try.html**. On the same page, you can also download a pdf template of the rods.

Reference

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An Interdisciplinary Approach to Problem Solving in the Mathematics Curriculum

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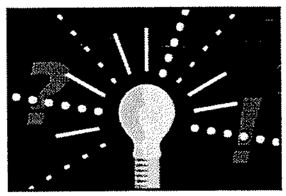

This article concerns an approach to teaching mathematics that facilitates the student seeing the connection between the process of solving nonroutine mathematics problems and solving significant meaningful problems in the student's life. The approach is embedded in a curriculum of nonroutine problems (curriculum in the sense of a process, guidelines, and a selection of sample problems versus a set sequence of lessons) developed and field-tested from 1989 to the present in secondary and undergraduate courses (London, 2007). In this article, I will briefly define a nonroutine problem, and discuss examples of nonroutine problems both mathematical and integrative in terms of their content focus.

Definition

For instructional purposes there are three essential characteristics of a nonroutine problem:

1. The problem requires three steps to complete: problem recognition and orientation, trying something, and persistence.
2. The problem allows for various solutions and requires students to evaluate a variety of potential strategies.
3. Every student is able to "solve the problem." Though the quality of solutions will vary, students will be able to confront the problem and generate a solution consistent with their ability and efforts.

In other works (1976, 1989, 1993, 2004, and 2007), I have discussed the theoretical and pedagogical basis for this definition, including arguing that the definition of the three steps of problem solving is consistent with other definitions of the steps of mathematical problem solving, including Polya's (1962), and that a properly structured curriculum of nonroutine problems is consistent with constructivist theory (Brooks, 1993). I have argued (1993, 2004, and 2007) that the ability to solve nonroutine