With the advent of legislation such as the current Individuals with Disabilities Education Act (IDEA), the inclusion of students with disabilities in schools has steadily increased. More recently, the level of language and cultural diversity represented in public schools has also increased. Consequently, today’s classrooms include students with a wide range of learning needs. For example, Carlos has a language-based learning problem, Ben struggles with attention problems, Maria’s weak reading skills interfere with her learning in all areas, and Jason has superior cognitive ability but great difficulty with mathematics. Classrooms rich in diversity most decidedly do exist—classrooms that give all students opportunities to learn about differences and abilities and about how to celebrate individuality while building communities. They can be wonderful learning environments for our children.

Today’s classrooms are certainly challenging environments for teachers. Designing lessons that are responsive to the unique learning needs of Carlos, Ben, Maria, and Jason, as well as the other eighteen students in the class, is not easy. Teaching mathematics in inclusionary classrooms can be daunting, especially when teachers are attempting to increase emphasis on the processes of doing mathematics (for example, communicating, reasoning, representing, connecting, and problem solving). It demands careful consideration of student learning attributes, appropriate levels of teacher support, and integration of teaching strategies that have been shown to have positive effects on student learning. This article describes the foundations underlying the process of instructional differentiation for students with learning problems in mathematics and illustrates its application using a lesson that emphasizes the five process Standards described in Principles and Standards for School Mathematics (NCTM 2000).

Characteristics of Special Needs Learners and Corresponding Teaching Strategies

Although each student has individual strengths and weaknesses, most children with learning problems share some common characteristics (Miller and Mercer 1997). Students with learning problems tend to be passive learners. Because of their history...
of difficulty with learning, they tend to attribute their successes and their failures to external, uncontrollable forces. They have little motivation to be active participants in learning because past experiences have reinforced their feelings of anxiety and low self-worth. These students need to be actively engaged in relevant learning situations that allow them to build and expand their conceptual knowledge while giving them the support to develop necessary underlying skills. They must be engaged in both “mistake making” opportunities, in which mistakes are valued for their learning, and structured opportunities to visually track their growth and progress. Simple visual displays, such as charts, can be wonderful “learning pictures” that help children see their learning progress. An example of such a visual display of learning is found in figure 5 and discussed later in the article.

Quite often, students with learning problems also have attention problems (Mercer and Mercer 2001; Miller and Mercer 1997). They may indeed attend, but they attend to the irrelevant details—concentrating on the colors of the shapes or the thickness of the lines instead of the differences in lines, angles, or formation. Students with attention deficits may have difficulty coming to or sustaining attention, resisting distractions, and controlling impulsivity. These students benefit from a structured, consistent environment in which clear expectations are communicated for learning and doing mathematics. Communicating clear expectations does not mean that the teacher must tell students how to do a task; rather, the teacher should give students a way to understand what is expected and a way to monitor their progress through a particular task. Giving students a cuing device to use as they complete an activity is one way to accomplish this. An example of a simple cuing device is a “goal sheet” that briefly states the goal of the activity and identifies the important steps, activities, or ideas the student will need to attend to while completing the activity. Figure 2b, which is discussed later in this article, shows an example of a simple cuing device. Cuing students in this way allows them to check off or cross through each idea as they complete it. This approach helps students attend to important ideas and promotes independent self-monitoring.

Students with attention problems often have difficulty with time management and transitions, but at the same time they benefit from a variety of opportunities to move and be physically engaged in
learning. Integrating visual organization into a lesson format and giving students opportunities to move and interact with their peers in structured situations are important to their success (Vaughn, Bos, and Schuem 2003). Clarifying expected behaviors for such activities is very helpful, especially for students with learning problems. Therefore, these students need to have relevant features of a task highlighted for them, to be given a variety of interactive activities during a lesson, and to have information visually organized for them.

Difficulty with memory is another common characteristic of students with learning problems (Mercer, Jordan, and Miller 1996). “Here today, gone tomorrow” seems to be the motto that they operate under. Explicitly (that is, purposefully and clearly) linking new information to prior knowledge and experience within relevant, authentic contexts allows students to “hook” new information to previously learned information, thereby facilitating the memory retrieval process (Mercer and Mercer 2001). Helping students link the old with the new can be accomplished through structured discussions in which both the teacher and the students engage in reasoning and communication. Teachers can help students with organization and retrieval strategies by giving them visual structure along with numerous opportunities to understand concepts and master skills through interactive, multisensory activities.

Language problems can also interfere with the learning of many students with learning problems. Language is used to mediate our world—to help us organize, categorize, reflect on, and respond to the input we receive. In order to integrate new concepts into existing schema, students must be able to link with prior knowledge and to translate the big ideas into language that makes sense to them (Kame ’enui et al. 2002). When students fail to see the links between concepts, mathematics becomes a rote exercise, and understanding remains at the algorithmic level. This type of learning can be detrimental, especially for those students who have difficulty with memory retrieval. Student learning is facilitated by reviewing previous concepts and explicitly demonstrating links in relevant, problem-solving contexts. In particular, vocabulary is often a barrier to students with learning problems. Discussing important vocabulary and using structured writing and guided questions to direct student reflection can help mitigate language problems.

Metacognition has to do with students’ ability to monitor their learning. It involves—

- evaluating whether students are learning;
- employing strategies when needed;
- knowing whether a strategy is successful; and
- making changes when needed.

These are essential skills for any problem-solving situation. Because mathematics involves problem solving, students who are not metacognitively adept have great difficulty with it. In fact, many students with learning problems in mathematics do not realize that students who are successful in mathematics employ strategies to tackle the mathematical tasks. Teachers must teach these students how to be metacognitive learners by helping them become aware of the strategies they are using and monitoring the effectiveness of the strategies they choose. Teachers who model and allow classmates to share problem-solving strategies, who reinforce students’ use of these strategies, and who teach students to organize themselves so they can access strategies will help students who have metacognitive deficits become metacognitive learners (Allsopp, Ingram, and Kyger 2000; Swanson 1999; Vaughn, Gersten, and Chard 2000).

Integrating Differentiation Strategies into a Lesson

Understanding the learning problems of our students and recognizing the teaching strategies that best facilitate their learning is important, but how does a teacher integrate these strategies into a whole-group lesson? The following lesson demonstrates how to accomplish this process. The teaching methods described here would benefit all children, yet are targeted at providing support to those children with learning problems. It is not our intention to suggest that these children may not need additional, individualized support. The modifications described here, however, will enhance the learning of students with learning problems within the context of a mathematics lesson taught in an inclusionary classroom. These modifications are designed to give all students in the classroom the opportunity to learn mathematics in a responsive and supportive environment.

The lesson, geared toward a second- or third-grade classroom, encourages students to build on their number sense to find alternate ways to add two-digit numbers. Differentiation strategies designed to support students with learning problems are indicated with italics. Table 1 summarizes this information for easier reference. (Students
### Table 1

**Instructional strategies designed to support students with learning problems**

<table>
<thead>
<tr>
<th>Lesson Sequence</th>
<th>Instructional Strategy</th>
<th>Purpose of Instructional Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Review Important Ideas</td>
<td>Review important vocabulary and symbols&lt;br&gt;Add vocabulary and representations (such as symbols and pictures) to math wall and “Math Talk” notebooks</td>
<td>• Make connections between old and new information&lt;br&gt;• Expand, reinforce language skills</td>
</tr>
<tr>
<td></td>
<td>Underline important information in word problems&lt;br&gt;Use color coding and arrows</td>
<td>• Focus on relevant information and important features of lesson</td>
</tr>
<tr>
<td></td>
<td>Use response cards (see fig. 1)</td>
<td>• Increase student risk taking and engagement</td>
</tr>
<tr>
<td>Establish Expectations</td>
<td>Use simpler, familiar problems&lt;br&gt;Use guided questioning and prompting (see fig. 2a)&lt;br&gt;Use cuing sheet (see fig. 2b)</td>
<td>• Increase student risk taking and engagement&lt;br&gt;• Make connections between old and new information&lt;br&gt;• Expand, reinforce language skills&lt;br&gt;• Promote independent student activity and reflection</td>
</tr>
<tr>
<td></td>
<td>Chart to categorize ways to add (see fig. 3)</td>
<td>• Model visual organization&lt;br&gt;• Reinforce metacognitive skills (such as the use of strategies)</td>
</tr>
<tr>
<td>Focus the Lesson</td>
<td>Partner and share&lt;br&gt;Have students come to board to show and explain work</td>
<td>• Expand, reinforce language skills&lt;br&gt;• Increase student risk taking and engagement&lt;br&gt;• Provide structured opportunities for movement</td>
</tr>
<tr>
<td></td>
<td>Use Strategy Sheet&lt;br&gt;Students can choose strategy from previous work and record strategies used (see fig. 4)</td>
<td>• Make connections between old and new information&lt;br&gt;• Reinforce metacognitive skills (such as the use of strategies)&lt;br&gt;• Promote independent student activity and reflection</td>
</tr>
<tr>
<td></td>
<td>Partner and share&lt;br&gt;Use color coding and arrows&lt;br&gt;Ask questions (teacher and students)</td>
<td>• Expand, reinforce language skills&lt;br&gt;• Increase student risk taking and engagement&lt;br&gt;• Focus on relevant information&lt;br&gt;• Reinforce metacognitive skills (such as the use of strategies)</td>
</tr>
<tr>
<td>Closure</td>
<td>Review by re-modeling strategies, using color coding and arrows&lt;br&gt;Summarize strategies, discussing similarities and differences&lt;br&gt;Review important vocabulary&lt;br&gt;Add vocabulary and representations (such as symbols, pictures) to math wall and “Math Talk” notebooks&lt;br&gt;Create visual display of learning progress (see fig. 5)</td>
<td>• Reinforce metacognitive skills (such as the use of strategies)&lt;br&gt;• Focus on relevant information and important features of lesson&lt;br&gt;• Make connections between old and new information&lt;br&gt;• Expand, reinforce language skills&lt;br&gt;• Increase student risk taking and engagement</td>
</tr>
</tbody>
</table>
should have access to materials such as counters, base-10 materials, ten frames, and number lines and should be familiar with their usage.)

**Review important ideas**

Before starting the lesson, help students connect their previous knowledge and experiences to it. One way to accomplish this is to review important vocabulary and symbols using color-coded visuals (for example, words, pictures, objects) to help students make connections between related ideas and to help students focus on relevant ideas. In the context of this lesson, the class can briefly review what addition means by having students indicate whether or not each of the problems displayed is an addition problem. An example is shown below. To help students focus on relevant information, you (or a student) can underline the important information as well as the question they are answering.

Myra has put 14 cupcakes in the box. Her mother gives her 4 more cupcakes to put in. How many cupcakes will Myra have in the box?

We are not advocating the use of clue words (such as more means add, less means subtract), because clue words do not indicate the same operation in every situation. (For example, consider this situation: Sam has 15 marbles. Keisha has 8 marbles. How many more marbles does Sam have than Keisha? More does not mean addition of 15 and 8 in this context.) You should encourage students to read the entire problem and attempt to make sense of the context instead of simply skimming for clue words and numbers.

Students can hold up index cards with an addition sign or a subtraction sign with the corresponding word to indicate which operation they think should be used. Color-code the written words and symbols as shown in figure 1 to help students with language problems. This method of communication helps students connect the symbol and word. It also can allow only you to see at a glance how students respond, which can encourage more risk taking from students. This gives you an efficient way to evaluate student understanding. After students have shown their cards, you can ask a volunteer to explain why the situation requires or does not require addition.

These color-coded vocabulary words and symbols can be added to a “math wall” in the class for students to reference when needed. Students can also develop “Math Talk” notebooks for individual reference. Organizing their notebooks by mathematics concept, problem-solving strategy, or another scheme will help students have a meaningful reference to mathematical language using their own words.

**Establish expectations**

The focus of the lesson is to encourage students to find alternate ways to add two, two-digit numbers. Therefore, the expectations for doing this task must be made clear and can be done by using simpler problems that are familiar. The familiarity can encourage risk taking (Malone and Lepper 1987), but it also allows students to begin to build connections to new ideas. Display two or three basic facts and ask students to work silently on this task: “If you did not know these already, show two ways you could figure them out.”

\[
\begin{align*}
8 + 5 \\
5 + 6 \\
8 + 9
\end{align*}
\]

Providing a structured form with guiding questions or prompts to direct student reflection (see fig. 2a) as well as a cuing sheet (see fig. 2b) can help students work independently. Show students how to write the addition statement on the form and where they can illustrate, with pictures or words, the strategy they used. Explain to students that this recording can be any kind of representation that will help them explain their reasoning to classmates later. For students with fine-motor or language problems, drawing pictures is an effective alternative for them to express their understandings. To give students additional opportunities to use language and to have structured opportunities for movement, have them work with a partner and share their solutions. (You can walk around while students are working so that you can later capitalize on various students’ approaches.)

After students have discussed in pairs, ask for volunteers to share one way to solve the problem. During the whole-class discussion, provide a visual organization for the students by drawing a chart (see fig. 3) to categorize the different ways to add. Provide structured opportunities for movement by having students come up and illustrate different ways to add. The headings should be added as students share the strategies they used.

As students share their approaches, make sure to model asking questions if students are not asking them (for example, “How did you do ____?” and “How did you know to do ____ first?”). Dur-
Figure 1
Color-coded index cards with words and corresponding symbols

Addition

Subtraction

Figure 2
Cuing forms that help students work independently

1. Problem: __________________________
2. Describe with pictures or words your way for solving the problem.

3. When I solved the problem, I got ________.

4. I used __________ to solve the problem.

(a) Structured form with prompts

Goal: To solve a mathematics problem

Checklist
You are to:

☐ 1. Write the problem at the top of the page.
☐ 2. • Use a strategy to solve the problem.
   • Use pictures or words to explain your strategy.
☐ 3. Write your answer in the blank.
☐ 4. Write the items or ideas you used to solve the problem.

(b) Cuing sheet for activity
ing this time, you can point out ways for students to make explicit to others their reasoning—for example, with different representations such as pictures, drawings, written explanations, and symbols. Students must understand that different purposes can exist for using different materials and representations in mathematics. One student may need to use manipulatives or a picture to reason through a particular mathematical problem. Another student may choose to use the manipulatives or picture to communicate his or her reasoning about the problem. Making these purposes explicit can often circumvent students’ resistance to using materials that they have somehow perceived only the “slower” students use. Continue asking for volunteers until you have exhausted the different approaches. You can facilitate understanding for students with learning problems by asking the class to look for and discuss similarities and differences between responses.

Focus the lesson
After the class has discussed this process of using alternate ways to add single-digit numbers, move to the focus of the lesson: finding alternate ways to add two-digit numbers. Display a problem such as the one below and ask students to think (silently!) about how they could solve this problem. Suggest that they might be able to use some of the strategies that were used with the basic facts. Suggest that other ways might exist. Students with learning problems may find success more easily by first choosing their “favorite” strategy from their prior work with facts. From this initial success, students with learning problems will be more apt to “take a risk” and explore other strategies.

Find two different ways to solve the following problem:
There were 37 Boy Scouts and 61 Cub Scouts at the meeting on Saturday. How many Scouts were at the meeting on Saturday?
To help students organize the information and provide visual structure, give students strategy sheets (figure 4 shows an initial strategy sheet) and a copy of the problem. Show them how to paste the problem to the top of the sheet, use the left column to record the strategy they used to solve the problem (remind students of illustrations used with the

<table>
<thead>
<tr>
<th></th>
<th>Ways We Know to Add</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting Up</td>
<td>Make a Ten</td>
</tr>
<tr>
<td>Start at 8. Count 9, 10, 11, 12, 13.</td>
<td>[Hand drawing]</td>
</tr>
<tr>
<td></td>
<td>[Diagram]</td>
</tr>
</tbody>
</table>

**Figure 3**
A chart of student-generated strategies provides visual structure for students.
basic facts), and write the numbers they added and their answer at the bottom. The key to the strategy sheet is to provide enough structure and cuing that allows students to independently make progress without doing the reasoning for them.

This type of sheet is beneficial for students and the teacher. For special-needs learners, the sheet provides a structure that allows them to proceed without constant direct guidance from the teacher. It also makes explicit for students the notion of using a strategy to tackle a mathematical task. For the teacher, the sheet provides insights into the level of students’ sophistication in the strategies they use—who is using a less sophisticated strategy, such as counting all or counting up, and who is using a more sophisticated strategy, such as compensating or making a ten. You can use this information to inform subsequent instruction, which may include providing additional, individualized support for the special-needs learner.

The ensuing discussion can follow the same structure as before, beginning with an opportunity to partner and share before the whole-class discussion. As students generate new strategies (such as adding the tens first), these strategies can be added to the strategy sheet and posted to the strategies listed on the math wall.

If a particular strategy does not emerge, you can pose a problem that exhibits the strategy. Consider the problem on the next page:

### Figure 4

An initial strategy sheet used by students to record strategies they use

<table>
<thead>
<tr>
<th>STRATEGY SHEET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paste problem here.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I solved the problem by:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting Chips or Base-10 Materials</td>
</tr>
<tr>
<td>Counting Up</td>
</tr>
<tr>
<td>Make a Ten</td>
</tr>
<tr>
<td>Other Strategy</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>_____ + _____ =</th>
</tr>
</thead>
</table>
Think about the following student’s work:

\[
\begin{array}{c}
48 \\
+36 \\
70 \\
+14 \\
84
\end{array}
\]

Is the student correct? What do you think the student did to get 84? Write down what you think about the student’s work.

After their responses have been discussed, students with learning problems greatly benefit from having the teacher review by re-modeling pertinent approaches. Using cuing techniques such as color-coding and arrows facilitates understanding for these students. Consider the same problem with such cuing integrated. Color highlights the major steps of the “add tens first” strategy, and arrows emphasize the relationship between the numbers:

\[
\begin{array}{c}
48 \\
+36 \\
70 \\
+14 \\
84
\end{array}
\]

**Closure**

To bring closure to the day’s activities, the class can review important vocabulary and summarize the different strategies used. As part of the summary, the class may want to compare the strategies for differences and similarities as well as efficiency. For example, you might say, “Let’s compare Jason’s approach to Maria’s approach. What similarities or differences do you notice?” New strategies can be added to the strategies on the math wall and Math Talk notebooks. Students can also create a visual display of their learning using a simple chart, such as the one in **Figure 5**. Such displays give students, teachers, and parents very concrete but powerful ways to see learning progress, especially when particular mathematics concepts and skills are practiced over several days or weeks.

**Concluding Remarks**

With the inclusion of specials-needs learners and with school mathematics no longer considered a body of isolated rules and procedures to memorize, teachers’ role in the mathematics classroom has changed. Becoming aware of the learning characteristics of special needs students and instructional strategies that directly impact these learning characteristics is an important first step in effectively teach-

**Figure 5**

Visual display or learning picture: Number of different two-digit addition strategies attempted

Name: Carlos
Concept/Skill: Two-Digit Addition Strategies

<table>
<thead>
<tr>
<th># of Different Strategies Used</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Successful</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unsuccessful</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Week 1 | Week 2 | Week 3 | Week 4 | Week 5

Strategy/ies used:
- **Red**: Add Tens, Make a Ten
- **Blue**: Use easier number, Compensation
ing mathematics to these students. These strategies can also be helpful for scaffolding the learning of the general education student who is unaccustomed to mathematics instruction aligned with the current reform. With these suggestions for differentiation, teachers can provide opportunities for all students to learn mathematics in meaningful ways.

References


