## Pick's Theorem

There is a function called Pick's theorem (named after the mathematician who discovered it) that enables you to find the area of any shape on the geoboard from the number of pegs on the perimeter of the shape (P) and the number of pegs inside the shape (I). The following sequence of activities suggests a way to find this theorem.

1. The square with an area of 1 has four pegs on its perimeter and zero pegs inside. Investigate other shapes with four pegs on the perimeter and zero pegs inside. Compare their areas. What about shapes with four pegs on the perimeter and one peg inside? Two pegs inside? Three? Four? Five? Any number? Write a formula that describes the relationship.
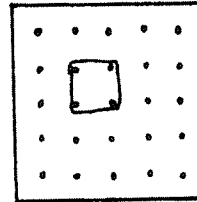
Remember: P = pegs on the perimeter
I = pegs inside
A = area of the shape

P	I	A
4	0	1
4	1	
4	2	
4	3	
4	4	
:	:	

2. Do the same investigation for shapes with other numbers of pegs on the perimeter.

P	I	A
3	0	
3	1	
3	2	
3	3	
:	:	

P	I	A
5	0	
5	1	
5	2	
5	3	
:	:	

P	I	A
6	0	
6	1	
6	2	
6	3	
:	:	

3. Now investigate patterns for the areas of shapes when the number of inside pegs stays constant and the number of pegs on the perimeter varies. For example:

P	I	A
3	0	
4	0	
5	0	
6	0	
7	0	
:	:	

P	I	A
3	1	
4	1	
5	1	
6	1	
7	1	
:	:	

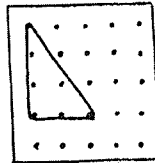| P	I	A
3	2	
4	2	
5	2	
6	2	
7	2	
:	:	

etc.

4. Finally, can you find a master formula that allows you to figure the area (A) for any combination of pegs on the perimeter (P) and pegs inside (I)? That's Pick's theorem.

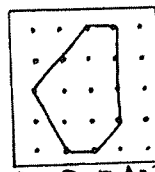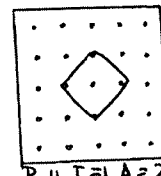Just to check your formula:

P=8 I=2 A=?   P=6 I=1 A=?   P=9 I=5 A=?   P=4 I=1 A=?

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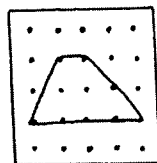
Worksheet 4.18

Make several shapes with no nails inside and three nails touching. Record the area. Next make shapes with four nails touching, and then with five nails touching. Fill in the rest of the first table. Do the same for the other tables. Do you see a pattern? Can you discover a rule for finding the area for each table?

1. Zero Nails Inside

Touching	Area
3	_____
4	_____
5	_____
6	_____
7	_____
•	
•	
•	
10	_____
T	$T \div 2 - 1$

Describe the rule in words:
Divide the number of nails touching by 2, then subtract 1.

2. One Nail Inside

Touching	Area
3	_____
4	_____
5	_____
6	_____
7	_____
•	
•	
•	
10	_____
T	_____

Describe the rule in words:
_____
_____
_____

Use your rules to answer these questions about shapes that cannot fit on the geoboard.

3. What is the area of a shape that has:
   a. 0 nails inside and 56 nails touching? _____
   b. 0 nails inside and 100 nails touching? _____
   c. 1 nail inside and 68 nails touching? _____
   d. 1 nail inside and 100 nails touching? _____

(continued)

From Building Success in Math (Langbort + Thompson)

4. Two Nails Inside

Touching	Area
3	_____
4	_____
5	_____
6	_____
•	
•	
•	
10	_____
T	_____

Describe the rule in words:

_____

_____

_____

5. Three Nails Inside

Touching	Area
3	_____
4	_____
5	_____
6	_____
•	
•	
•	
10	_____
T	_____

Describe the rule in words:

_____

_____

_____

6. What is the area of a shape that has:

  a. 2 nails inside and 74 nails touching? _____

  b. 2 nails inside and 100 nails touching? _____

  c. 3 nails inside and 54 nails touching? _____

  d. 3 nails inside and 100 nails touching? _____

This chart consolidates the information from Worksheet 4.18. Fill in all the blanks. Make the shapes on the geoboard to help you. Are there patterns to help? Can you state a general rule?

$\frac{1}{2} + 1$

Nails Touching

Nails Inside	3	4	5	6	7	8	9	10
0	½	1	1½	2	2½	3	3½	4
1	1½		2½	3		4	4½	
2	2½		3½			(5)	5½	
3		4	4½			6	6½	7
4			5½			7	7½	
5			6½			8		
6			7½			9		
7						10		

Use your general rule to answer the following questions. What is the area of a shape that has:

1. 25 nails touching and 82 nails inside? _____

2. 57 nails touching and 100 nails inside? _____

3. T nails touching and I nails inside? _____