> *Nonproportional thinker 3:* "Their paint mixtures were exactly the same. The paint mixtures are the same because even though it said Nancy used more blue tint than Kathy, it also says Nancy mixed in more white paint than Kathy. This would make the mixtures seem the same."

In discussing this task and others with teachers in these professional development sessions, team members had teachers share their thinking and solution methods without making any judgments about the correctness of the teachers' solutions or thinking patterns. This approach allowed everyone to explore the relationships involved in the problems and further develop their abilities to think proportionally. By reflecting on their own reasoning, as well as on samples of students' reasoning, teachers (a) came to a better understanding of their own thinking, including their sometimes erroneous thinking; (b) learned the fundamental differences in proportional and nonproportional reasoning; and (c) became aware of the power of simple proportional reasoning tasks in stimulating students' thinking.

Proportional Reasoning Activities across the Middle School Mathematics Curriculum

ANOTHER TYPE OF ACTIVITY IN THE MODULE required teachers to solve proportional reasoning problems from one or more of the following mathematics content areas: number and computation, geometry and measurement, probability and statistics, and algebraic reasoning. The purpose of these activities was to demonstrate the pervasiveness of proportional reasoning in the middle school mathematics curriculum and to provide teachers with the opportunity to apply proportional reasoning in these contexts. The following activity was closely linked to measurement:

> You are the ace detective for a local law enforcement agency. You have been called in to help solve a case. Suspects have been narrowed to 3 people. One is 5 feet tall. The second is 6 feet tall. The third is 7 feet tall. The only clue left from the scene of the crime is a handprint. Use the handprint to help narrow the list of suspects. [A handprint that had a span approximately one-third greater than that of a typical adult who is 6 feet tall was shown with the problem.]

Teachers discussed possible solution methods in small groups and shared them with the entire class. This discussion usually led to ideas about collecting data from the class members about their hand spans and related heights. The data were collected and entered into graphing calculators or spreadsheets structured as shown in **figure 1**. Teachers then analyzed the data and decided how to use the ratios in column C along with the span of the distributed handprint to predict the height of the suspect. Some groups decided to find and use the mean of the ratios in column C, and others decided to use the median of those ratios. The teachers then found the predicted height of the suspect by multiplying the span of the handprint by the mean or median of the ratios.

	A	B	C
1	HEIGHT	HAND SPAN	RATIO
2	<value>	<value>	=A2/B2
3			

Fig. 1 Spreadsheet format for detective activity

The team led a discussion that focused on how proportional reasoning was involved in this problem. Usually, comments centered on the idea that the mean or median ratio, height/hand span, represented the person's height in terms of his or her hand span; such comments reveal multiplicative reasoning. The predicted height of the suspect could, therefore, be found by multiplying the mean or median ratio and the hand span of the suspect. Further discussion focused on the use of proportional reasoning in the profession of forensic medicine.

Without a doubt, one of the most intriguing and provocative activities in the module was "What Time Is It?" (Gallin 1999) (see **fig. 2**). For this task, the teachers were shown a picture of a clock face having both an hour hand and a minute hand set in specific positions. The intriguing part of the task was that the clock face had no numerals, and the 12 did not necessarily belong at the top.

The task required teachers to determine the correct time by using the relationship between the minute

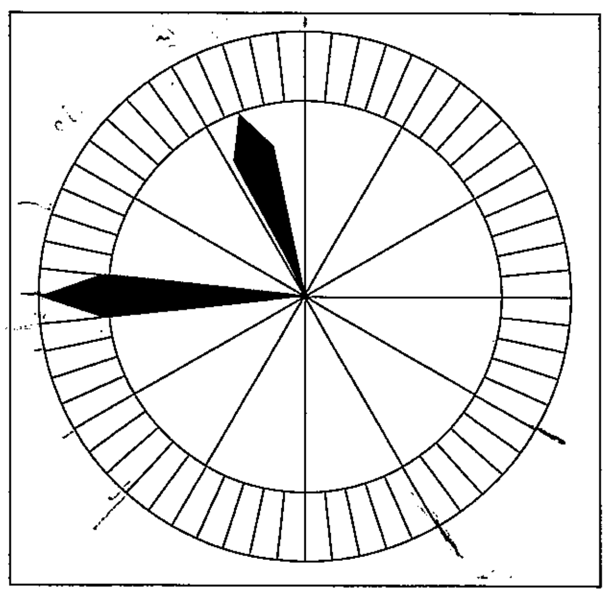


Fig. 2 "What Time Is It?" (From Gallin [1999])

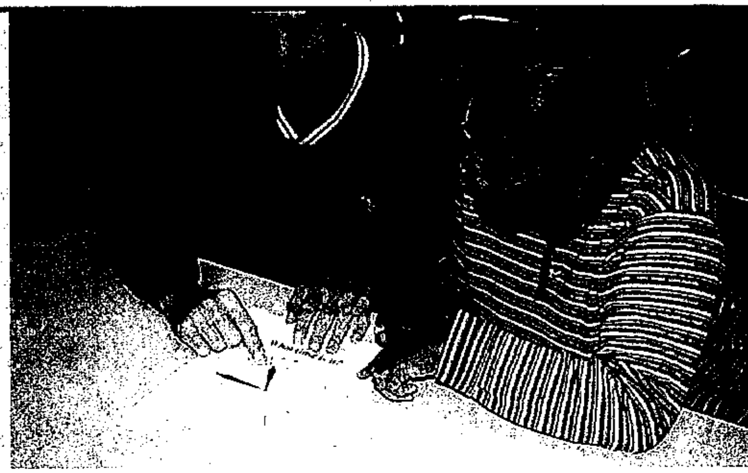
hand and the portion of an hour that it had traveled around the clock face, along with the hour hand and the portion of an hour that it had traveled from one of the hour marks on the clock to the next hour mark. In fact, these two relationships could be represented as equal ratios—one of the most frequently accepted definitions of proportional reasoning.

After teachers worked on the problem and shared their thinking, several ideas became clear. The most common solution method was to rotate the clock to various positions, mentally place 12 at the top each time, determine what fraction of the total trip around the clock the minute hand had completed, and check to see if the hour hand had completed the same fraction of the trip between the two hour numbers on both sides of it. A number of teachers were surprised to learn that the movement of the hour hand was so precise, but all were excited about the task and thought that it would be interesting, challenging, and motivating for their students.

In the context of each of these problems, the team members and the teachers discussed the differences between solving proportions and thinking proportionally. All agreed that proportional reasoning is not a well-developed curricular topic and that attention is normally given to solving proportions procedurally by using cross products. The teachers explored other methods of solving simple proportions by using proportional reasoning, such as working with unit rates and factor of change.

Follow-up Sessions during the Subsequent School Year

DURING THE SUBSEQUENT SCHOOL YEAR, THE team members held a six-hour professional development session addressing proportional reasoning. At this session, the team focused on experiences of the teachers when they taught proportional reasoning in their classrooms. Each teacher brought an activity using proportional reasoning to share, along with students' work samples. This sharing opportunity led to spirited discussions, and teachers were keenly interested in what others had done. The team also presented new activities involving proportional reasoning. One of the most interesting of these activities used elastic bands, which can be obtained at a sewing store, each marked by the user into 100 equal subdivisions to model percents and decimals. By stretching a marked band to match the length of a person's arm from shoulder to fingertip, for example, the user can see that the hand is about 25 percent of the arm's length. In effect, the bands model the meaning of *percent;* they divide an object into 100 parts to help the user determine the number of parts (the percent) corresponding to a certain component of the object (AIMS Education Foundation 2000).



Outcomes

ASSESSMENTS REVEALED THAT TEACHERS IMproved their own proportional reasoning skills, they learned more about student thinking, and they learned important applications that require proportional reasoning. One teacher's comments during the subsequent school year serve as testimony to the value of the professional development sessions: "My work with the students, after using some specific proportional reasoning activities from the workshop, yielded much better products. The students were probing and looking for their own meaning, as opposed to simply looking for the right answer. And, to my surprise, I found that many of them were not satisfied with just the right answer but wanted to share how they got there."

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