Algebra: It's Elementary!

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For most of the twentieth century, the study of algebra was reserved for those students who had successfully completed the study of arithmetic and were entering high school. Algebra was considered too abstract for elementary and middle school-age students. There were three studies and documents that had a profound effect on this approach:

1. In the late 1980s, the College Board commissioned a study of the relationship between students' high school course-taking histories and their successful entrance into and completion of college (Pelavin and Kane 1990). Results showed that students who had completed Algebra I were most likely to complete four years of college.

2. In the 1989 *Curriculum and Evaluation Standards in School Mathematics* and again in the 2000 *Principles and Standards for School Mathematics (PSSM)*, the National Council of Teachers of Mathematics (NCTM) identified algebra as a curricular strand, with instruction in algebra to begin in the elementary school mathematics program and extend through grade 12. NCTM also pointed out the essential nature of algebraic reasoning and of algebraic concepts to success in the study of other strands of the mathematics curriculum (for example, number and operations, measurement, and probability) and to success in other disciplines such as the sciences and geography.

3. *PSSM* and a National Task Force report stressed the importance of algebraic reasoning to successful performance in jobs in the 21st century.

In response to these findings and recommendations, school systems nationwide began requiring all students to enroll in Algebra 1, and to do so no later than grade 9. So most school districts made enrollment in Algebra 1 mandatory for all of their grade 8 students.

Despite the recommendations for earlier instruction in algebra, most elementary school mathematics programs did not include algebra as a major content strand. As a consequence, great numbers of students had difficulty with algebra in grade 8. Most were unable to make the transition from instructional programs that focused on arithmetic thinking to those that required abstract algebraic reasoning (Greenes and Findell 1989/1999). For several years, organizational techniques that might facilitate success with Algebra 1 were explored, such as slowing the pace of instruction by offering Algebra 1 over a two-year period or adding tutorial/discussion sections that were offered in tandem with Algebra 1. For most students, these techniques were not effective. What was needed was a curriculum that would provide a continuum of instruction in algebra, beginning in prekindergarten and extending through grade 12.

Before describing a program for students that enables them to get a handle on the big ideas of algebra, let us consider why algebra is such an important part of the study of mathematics.

Why Is Learning Algebra Such a Big Deal?

Algebra is sometimes referred to as generalized arithmetic because it formalizes arithmetic relationships. Its power lies in the ways it allows us to represent relationships among quantities, to describe properties of operations (such as commutative and distributive), and to describe patterns. Algebra provides rules for manipulating symbols, such as simplifying an expression and then solving for an unknown. Reasoning algebraically is essential to the study of other domains of mathematics, including geometry, measurement, probability, and data analysis. Algebraic reasoning is also needed in the sciences, the social sciences, and the arts. While in the past, algebra instruction was reserved for older students and focused primarily on the manipulation of symbols and the solutions to equations, today's algebra instruction should begin with the youngest students and focus more on the "big ideas" of algebra and on reasoning algebraically.

Understanding the "Big Ideas"

For students in the elementary grades, there are three big ideas of algebra to tackle. Although these big ideas are individually identified below, they often appear in problems in combination. Sample problems are given to show application of the big ideas.

Big Idea 1: Variables

A variable can be an object, a geometric shape, or a letter that represents a number of things. Variables are used in three ways in elementary school mathematics. They are used to represent unknowns, to represent quantities that vary, and to generalize properties.

◆ Representing Unknowns

Unknowns are variables that don't vary! That is, they have fixed values. Variables with fixed values are used extensively in equations in the early grades, for example, $3 + 2 = n$ and $t + 3 = 7$. Students learn that the same letters or shapes in equations represent the same values.

Sample Problem

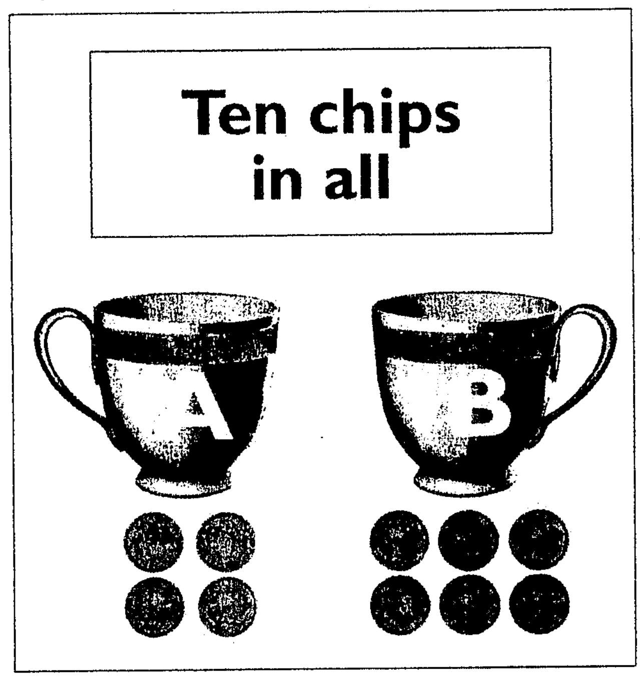
Students are shown two pictures (see picture at the bottom of this page). In one picture there are two identical toy cars with a price tag of $2 for both toys. The second picture shows the same toy car and a toy dog with a price tag of $5 for both toys. This problem is their introduction to solving systems of two equations with two unknowns; the two equations are car + car = $2 and car + dog = $5. The unknowns are the price of the toy car and the price of the toy dog. By the time students reach grade 5, they learn how to solve word problems that involve first writing equations to represent the mathematical relationships and then solving for the values of the variables.

◆ Representing Quantities That Vary

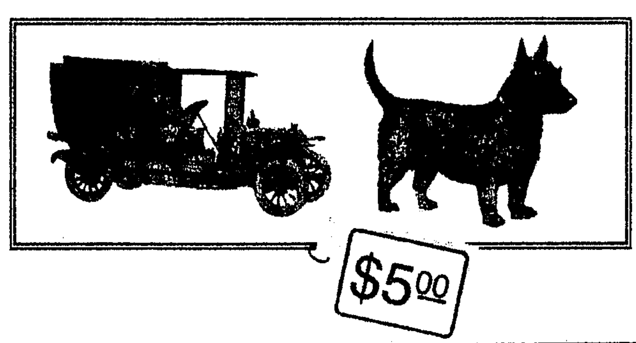
In some types of equations, the variables are not fixed. The variables can assume different values. Students can be introduced to the concept of variables as representing varying quantities as early as the prekindergarten level.

Sample Problem

Students are presented with two cups—let's call them A and B—and 10 chips. Students are asked to find all of the different ways that they can place the chips into the cups.



This problem can be represented algebraically by writing $A + B = 10$. A and B vary because they can have lots of different values. For example, if $A = 0$, then $B = 10$; if $A = 1$, then $B = 9$; if $A = 2$, then $B = 8$; and so on. Not only



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do students learn that there are 11 different pairs of values that solve the equation, but they also learn that once they choose a value for A, the value of B is determined. Similarly, if they select a value for B, then the value of A is determined.

Variables that take on different values are found in all formulas. Consider the formula for the perimeter of a rectangle. In the formula $P = 2l + 2w$, the values of l and w may be different for different rectangles. Likewise, in the formula $D = rt$, the rate and time will vary for different situations.

◆ Generalizing Properties

Early in their mathematics education, students are introduced to special relationships that exist among sets of numbers under specific operations. When students learn about these properties, the number of facts that need to be remembered is reduced, and computations are simplified.

Some Properties

If students learn that $3 + 4 = 7$, then they also know that $4 + 3 = 7$, because for addition of whole numbers, when you change the order of the addends, the sum remains the same. This rule, the Commutative Property of Addition of Whole Numbers, holds true for all pairs of whole numbers, so it can be represented using variables as $a + b = b + a$.

Likewise, $n \times 1 = n$ is the Identity Property of Multiplication of Whole Numbers. It states that when any whole number, let's call it n, is multiplied by 1, the product is the whole number, n. Using variables provides a shorthand way of describing relationships that apply to all numbers in a set.

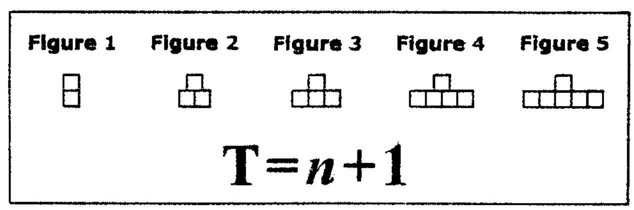
Big Idea 2: Patterns and Functions

In the early grades, students are exposed to two different types of patterns: those that repeat and those that grow or shrink. A repeating pattern is one in which a group of elements repeats, like BBBCBBBCBBBC. . . A growing or shrinking pattern is one in which elements do not repeat. Rather, there is a change between successive elements, and every element in the pattern is related to the preceding element in the pattern in the same way. An example of a growing pattern is 1, 3, 5, 7, 9, . . . ; each element in the pattern is two more than the preceding element. An example of a shrinking pattern is 100, 90, 80, 70, . . . ; each element in the pattern is 10 less than the preceding element. Growing and shrinking patterns lead to generalizations and to representations of the generalizations using variables.

Children learn to describe growing and shrinking patterns in words and to predict what comes next. The growing patterns may be made from pictures or from three-dimensional objects, like cubes, toothpicks, or buttons.

Sample Problem

Show the student a pattern of constructions made from cubes in which Figure 1 in the pattern shows one cube on top and one cube on the bottom, Figure 2 shows one cube on top and two cubes on the bottom, Figure 3 shows one cube on top and three cubes on the bottom, and so on. (See example below.) The student might describe the pattern by saying, "The number of cubes on the bottom is the same as the number of the figure. There is always one cube on top." When they reach grades 4 and 5, students can use symbols to write the function that shows the relationship between the total number of cubes (let's call it T) and the figure number (let's call it n). The function would be $T = n + 1$.



Students also identify patterns when they complete function tables. They learn that for every input there is exactly one output. In the early grades, students complete function tables when the rule is given, such as "Add 3." When students enter later grades, they not only complete function tables when the rule is given, but they also have to collect data, organize the data in a table, figure out the function, and record the function using symbols.

Big Idea 3: Proportions and Proportional Reasoning

A proportion is a special type of function in which the relationship between quantities is multiplicative. Reasoning about quantities that are related by multiplication is referred to as proportional reasoning.

In elementary school mathematics programs, proportional reasoning occurs everywhere. In the study of place value, students learn that 1 ten is equal to 10 ones, so they figure that 2 tens (2 x 10) is equal to 20 ones, 3 tens (3 x 10) is equal to 30 ones—each regrouping involves multiplication. In the study of measurement, when students know that there are 4 cups in 1 quart, they can reason that there must be 8 cups (2 x 4) in 2 quarts, and 12 cups (3 x 4) in 3 quarts. In this process, the student is reasoning proportionally. When students construct equivalent fractions and when they use the key on a map to figure out actual distances between cities, they are reasoning proportionally.

In grades 4 and 5, students learn to record proportional relationships using symbols and to graph the relationships. The graph of a proportion is a straight line that contains the origin. A strong foundation of working with proportions and proportional reasoning sets the stage for the exploration of other types of functions at the middle and high school levels, functions whose graphs do not contain the origin.

But Can Children Do Algebra?

Recent research studies with elementary-school-age youngsters have shown that children in grades 1–3 have great interest in solving algebraic problems because they like the mystery of working with unknowns and having to reason their way to the solution (Tsankova 2003). Furthermore, they have no difficulty working with different representations of variables; they actually thought that using letters for variables was much easier than drawing geometric shapes or using objects. The same results were found for students in grades 4 through 6 (Dobrynina 2001). What was astounding was that without instruction, 15 to 20 percent of students in the early grades and 22 to 27 percent of students in grades 4 through 6 could reason algebraically and solve systems of equations. Clearly students have informal knowledge of some algebraic ideas, and they are ready to explore these and other algebraic concepts and skills.

The Role of the Teacher

There is a strong connection between arithmetic and algebra that is important for the teacher to emphasize. There are also connections to other core areas of the mathematics curriculum, particularly measurement. Helping students make these connections will help to solidify their understanding of the big ideas of algebra, as well as key concepts of the other strands. There are more and more instructional materials that can be incorporated into the regular program and that enable students to gain sufficient understanding of the key ideas of algebra in order to help ensure success in the later formal study of algebra. The job, of course, is finding those resources. When examining supplemental instructional materials, be sure that the key ideas described in this article are addressed and that there is a clear development of ideas across grades. Also, be sure that the activities for students require them to describe their thinking, document their solution steps, and provide rationales for their solution approaches.

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